MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

PHILOSOPHICAL ANALYSIS OF WAVE-PARTICLE DUALITY OF PHOTONS

A THESIS TO BE SUBMITTED TO
MANIPAL ACADEMY OF HIGHER EDUCATION

FOR FULFILMENT OF THE REQUIREMENT FOR
THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY

BY
VARUN S BHATTA

UNDER THE GUIDANCE OF
Dr SUNDAR SARUKKAI

SCHOOL OF HUMANITIES
NATIONAL INSTITUTE OF ADVANCED STUDIES
BENGALURU, INDIA

MANIPAL
ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

DECLARATION BY THE CANDIDATE

I declare that this thesis, submitted for the degree of Doctor of Philosophy to Manipal Academy of Higher Education, is my original work, conducted under the supervision of my guide Dr Sundar Sarukkai. I also wish to inform that no part of the research has been submitted for a degree or examination at any university. References, help and material obtained from other sources have been duly acknowledged.

Candidate

Varun S. Bhatta
Registration number:143600101

Date: 05 December 2020

MANIPAL
ACADEMY *of* HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

CERTIFICATE

This is to certify that the work incorporated in the thesis "**Philosophical Analysis of Wave-Particle Duality of Photons**" submitted by Varun S. Bhatta was carried out under my supervision. No part of this thesis has been submitted for a degree or examination at any university. References, help and material obtained from other sources have been duly acknowledged.

Research Guide

Dr Sundar Sarukkai
Former Professor
School of Humanities
National Institute of Advanced Studies
Bengaluru, India

Date: 05 December 2020

Contents

1 Nature of Scientific Entities **1**
1.1 Introduction to Metaphysics of Entities 2
1.2 Nature of Theoretical Entities 4
1.2.1 Theoretical Entities: Ontology of a Theory 4
1.2.2 Theoretical Entities: Constitution by Law-like Principles 10
1.2.3 Theoretical Entities: Some Concerns 11
1.3 Nature of Unobservable Entities 12
1.4 Disinterest about Physical Entities 15
1.4.1 From Entity-realism to Anti-realism 15
1.4.2 Transition from Material to Physical 16
1.4.3 Problems Concerning Quantum Entities 19
1.4.4 Naturalisation of Philosophy 23
1.4.5 Rise of Structuralism 24
1.5 Plan of the Thesis 26

2 Nature of Light and Photons **32**
2.1 The Passage of Light through Ages 32
2.1.1 Ancient and Medieval Knowledge Systems 32
2.1.2 Mechanical Interpretations of Light 35
2.1.3 Electromagnetic Interpretation of Light 37
2.1.4 Quantum Mechanical Interpretations of Light 38
2.1.5 Field Interpretation of Light 40
2.1.6 The Age of Photons 42
2.2 Interpretations of Photons 46
2.2.1 Notions of Light-quantum in Dirac's Analysis 46
2.2.2 Ontological Status of Photons 47
2.2.3 Operational Interpretations of Photons 49

3 Different Notions of Duality **52**
3.1 Basic Forms of Duality 52
3.2 Duality in Logic 55

3.2.1 Duality between Truth-Functions 55
3.2.2 Generalized Procedure 58
3.2.3 Duality and Equivalence 59
3.2.4 Duals of Duality 60
3.3 Duality in Projective Geometry 60
3.3.1 Duality of Points and Lines 60
3.3.2 Principle of Duality 61
3.3.3 Duality and Formal Geometry 63
3.4 Duality of Vector Spaces 67
3.4.1 Dual and Second Dual Spaces 67
3.4.2 Duality of Objects or Structures? 69
3.5 Duality of Groups 71
3.6 Duality of Categories 73
3.7 Dual Theories of Physics 74
3.8 Duality in Other Branches of Science 76
3.9 Duality and Symmetry 76

4 Wave-Particle Duality and Light **80**
4.1 Plurality of Wave-Particle Duality 80
4.1.1 Introduction 80
4.1.2 Multiple Historical Interpretations 82
4.1.3 Current Plural Views about Duality 84
4.1.4 Drawbacks of Unified Narrations 88
4.1.5 Nature of Plurality 92
4.1.6 Conclusion 94
4.2 Duality and Physical Interpretations of Light 96
4.2.1 Planck's Law and Initial Analyses 96
4.2.2 Einstein's Analysis of Light 105
4.3 Applicability of Classical Concepts for Light 110
4.4 Equivalence of Wave and Particle Interpretations 112

5 Critique of Wave-Particle Duality of Single Photons **116**
5.1 Quantum Optical Analysis of Photons 117
5.1.1 Difference between Classical and Quantum Optics 117
5.1.2 Experiments Concerning Duality of Photons 118
5.1.3 Analysis of AC 120
5.1.4 Analysis of MZ 122
5.1.5 Summary of Analysis 124
5.2 Duality Claims about Photons 125

5.3 Critique of Duality Based on Complementarity 127
5.4 Nature and Evaluation of Analogies 130
5.5 Critique of Particle Analogy 132
5.6 Critique of Wave Analogy 137
5.7 Conclusion 140

6 Controversy About Interference of Photons 141
6.1 Confusion about Interference 141
6.2 Superposition and Interference in Classical Optics 144
6.3 Dirac's Interpretation of Interference 147
6.4 Modern Interference Experiments 150
6.5 Controversy about Interference of Photons 156
6.6 Sources of Controversy 159
6.7 Conclusion 163

7 Conclusion 164

Bibliography 166

Acknowledgements

The submission of this thesis marks an important milestone in the journey that started several years back. Embarking on this journey and traversing the last nine years has been both challenging and fulfilling. Reaching this landmark would not have been possible without the good intention and help of several people.

Even though I was interested in philosophy right from my bachelors in engineering, it was only few years into work, I started entertaining the idea of doing academic research. I would not have made up my mind to pursue philosophy if Sundar — among various philosophers to whom I had written enquiring (rather overconfidently) about the prospective of doing philosophy — had not shown interest to reply. The correspondence with him over several conversations played an important role in making the decision. However, it was the establishment of Manipal Centre for Philosophy and Humanities (MCPH) that eventually motivated me to take the final plunge. In retrospect, I would not have been writing this acknowledgement if these series of events would not have happened. Sundar's role does not end there. Even though his unique pedagogy, along with the other faculty at MCPH, guided me throughout the master's programme and the research fellowship, it was during the doctoral research that a deeper engagement with him was possible. The last six years of training under him has shaped my style and attitude of doing philosophy. The research carried out in this thesis is an attempt to put into practise what I have learnt from him. Thus, the positive aspects of the thesis should be credited to his teaching and perseverance.

The doctoral research happened in two different institutions: the first two years in MCPH and the remaining four years in National Institute of Advanced Studies (NIAS). At MCPH, I am thankful to the faculty — George Varghese, Apaar Kumar and others. I thank Meera Baindur for her continued support throughout this phase. I am grateful for the support and warmth shared by Anindita Majumdar and Mrinal Kaul. I acknowledge the solidarity provided by the then PhD colleagues at MCPH — Madhava Chippali, Tapaswi H M, Srajana Kaikini and others. The support provided by the office and library staff has been of great help. Lastly, I am grateful for the company of Kaushik Ramu, Pooja Nayak and Sammitha Sreevatsa throughout this period.

At NIAS, I found new set of academic support. I appreciate the effort shown by Dr Baldev Raj, the previous director of NIAS, for making the transfer from MCPH and NIAS possible. I sincerely thank Anindya Sinha and Arnab Rai Choudhuri for being part of my doctoral advisory committee. The support provided by NIAS doctoral committee has been helpful at various stages. Here, the enduring aid provided NIAS PhD coordinator J N Sandhya cannot be forgotten. During the NIAS stint, various fellowships have supported my research. I am grateful to Sutasoma Trust (London) and Philosophy-in-Action project (funded by Tata Trust) for providing the much needed financial support. I also thank Indian Council of Philosophical Research (ICPR) for providing JRF for two years period. The final phase of the research at NIAS has been free of hindrance due to the support of the library staff Hamsa Kalyani and Vijayalakshmi, NIAS office, the canteen staff and the untiring gardeners. Lastly, I would like to thank my colleagues and friends — Savitha Suresh, Krupa R, Anupama Mahajan, Leya Matthew, Surya Sankar Sen, Siddharth S, Nitesh Anchan, Vijay Govind Nath, Mansi Rathour and others.

During my research at both MCPH and NIAS, through the opportunities provided by several workshops and conferences, I was able to interact with various people. I acknowledge that K V Akshara and Raghurama Raju have been a constant source of inspiration. The interactions with N S Gundur and Srinivas Ramanujam have enriched me in numerous ways. I thank Aparna Uppaluri for making me part of the philosophy of science workshop at NCBS. I fondly remember the conversations with Anil Ananthaswamy on both of our research topics. I thank Anna Agathangelou and Gordon McOuat for providing the opportunity to visit Toronto to attend a relevant conference. I am specially indebted to Gordon for not only taking good care of me at Toronto and spending time to discuss my research, but also for connecting me to other Canadian philosophers of science. Here, I have to thank Arun Bala, Joseph Berkovitz, Markus P. Mueller, Jagdish Hattiangadi for taking time and sharing their thoughts about my initial research plans. I also thank the thesis examiners for taking the time and going through the thesis meticulously and providing their comments. These queries have helped me patch the gaps that I had overlooked, and thus, has made the thesis less erroneous and less confusing than before.

None of what I have been able to carry out would have been possible without the love and care shown by my family and friends. I respect the unconditional support provided by my parents Vijaya and Sathyanarayana and I dedicate this thesis to them. I thank my sister Smitha and brother-in-law Shivarama for having supported me in various ways. I thank my in-laws Sreevatsa, Veena and Samyamee for their understanding and providing me the space and time to work on my thesis. I am glad for the company of several friends — Mridula and Sumanth; Kaushik, Pooja and Shireen; Mrinal, Anindita and Bhushan — for showing interest in my work and making this phase less lonely. Finally, I am grateful to Sammitha without whose presence this journey would not have been this peaceful and meaningful.

Chapter 1

Nature of Scientific Entities

This thesis is a study about the insights and challenges provided by the concept of wave-particle duality regarding our conceptions of physical entities. Even though varied kinds of entities are part of our common-sense reality, understanding the nature of these — as several metaphysical queries show — is not straightforward. This task becomes even more arduous in science, especially in the context of physics since it deals with entities that are not directly observable and are completely illuminated through their respective theories. Because of these and other factors, there are several open questions pertaining to the ontology of physics. It is in this larger exploration that the present study is situated. In the thesis, I will specifically analyse photons in the context of wave-particle duality experiments and show the invalidity of the concerned duality claim. I will also point out the controversy surrounding the concept of interference for photons and identify the reasons for this confusion.

An appropriate place to start this exploration is by getting acquainted with the notion of *scientific entities*. In section 1.1, I start with a brief introduction to few important metaphysical questions regarding entities in general and the category of scientific entities. Among the various ways of characterising scientific entities, I focus on their unobservability and the relation between entities and theory since these aspects play a decisive role in the case of photons. I explore these aspects of scientific entities in sections 1.2 and 1.3. After this introduction to scientific entities, I present the current dominant trends about the study of physical entities in both modern physics and philosophy of science. As I will substantiate in section 1.4, entities of physics transitioned from being "material" to "physical". Along with this, the ontology of physical entities faced strong criticisms due to the development of quantum mechanics and from structuralist stances in modern analytic philosophy of science. It is important to discuss the current disinterest in physical entities to motivate the topic of the present thesis — study of physical entities in general — and the specific mode of analysis I have adopted in the thesis. I will discuss these aspects and the plan of the thesis in section 1.5.

1.1 Introduction to Metaphysics of Entities

The diverse kinds of entities around us have been the perennial subjects of philosophical enquiries. Here, "entities" refer to the general category of things that exist.[1] There are various important philosophical questions pertaining to entities. Among these, a fundamental one is about ontology: what all kinds of entities exist? Do only ordinary things like tables and sand particles should be the preferred candidates here? Or do processes and phenomena like waves and grins[2] also exist? And, similar to concrete things which we perceive around, do abstract entities (Rosen 2018), possible objects (Yagisawa 2018), fictional entities (Kroon and Voltolini 2018) exist?[3] Apart from the larger ontological queries, there are other metaphysical explorations that have been crucial for understanding the nature of entities. One of these is regarding the nature of existence itself. For instance, the characteristic of concrete entities' existence has been interpreted in several ways. Contrary to Russell's proposal that existence has to be a second-order property (Nelson 2019), the classical Indian realist school Nyāya-Vaiśeṣika (henceforth NV) argues that existence is the most general category and material things belong to this category (Matilal 1990, 272). Aristotelian system interpreted the nature of existence of these things through the theory of hylomorphism (Lowe 1998; Shields 2013). Medieval European mechanical philosophers, like Descartes, articulated material things' existence through the criterion of spatial extension (Garber 1992).

Related to the above set of enquiries, another concern has been regarding the mereological nature of material entities: how can numerous things come together to constitute an individual entity?[4] This compositional characteristic of material entities has given rise to various philosophical questions, some of which are: the problem of many (Weatherson 2016), Sorties paradox (Hyde and Raffman 2018), the nature of temporal parts (Hawley 2018) and the possibility of eliminativism (Wasserman 2018; Ramsey 2019). These queries along with others — like that of persistence through change — have motivated the development of the concept of substance (Bhaduri 1946, 22; S. H. Phillips 1997; Robinson 2018). Since not all kinds of material entities — for instance, various stuffs like lump of clay, drop of water — seem to qualify as substances, modern Western analytical philosophy distinguishes *objects* from general *entities*. Unlike the latter, every kind of

1. The presupposition of this general category can be observed in several Indian and Western analytic philosophical works. For instance, Matilal (1990) uses the concept of entity to refer to variety of things. Lowe (1998) also uses "entities" category. However, S. H. Phillips (1997) uses "existents" instead.

2. Lowe (1998, 37) provides these examples.

3. There have been numerous responses to this question about ontology. For the ontological stances of various classical Indian philosophical systems, see Matilal (1990), Halbfass (1992) and S. H. Phillips (1997). For ontological proposals in early modern and modern western philosophy, see Moore (2012) and Thomasson (2019).

4. In the Indian context, for an introduction about the arguments between NV and Buddhism schools about wholes and parts, see Matilal (1986, 5). For an introduction to mereology in the modern Western analytical tradition, see van Inwagen (1995) and Varzi (2019).

object possesses a criterion of identity (Wiggins 1967; Noonan and Curtis 2018) and a principle of individuation (Gracia 1988; Lowe 2003). In Indian philosophical traditions, especially for NV, the notion of individuals has also been central (S. H. Phillips 1997, 47).

The above discussion attempts to capture some of the important metaphysical questions pertaining of entities. Since scientific entities are a subset of this general category, the above queries are equally relevant for their understanding. However, before these questions are dealt in detail specifically with regard to scientific entities, it is necessary to briefly talk about this category: how to demarcate scientific from non-scientific entities?[5] There have been various responses to this question. A pragmatic and straightforward answer is that "scientific entities" are those which are the subjects of science as a discipline. It is crucial to acknowledge the disciplinary aspect of scientific entities because, as Daston (2000b, 2) points out, some of these entities are unlike ordinary, quotidian things that "throw themselves in front of us... they do not need to be discovered or investigated; they possess the self-evidence of a slap in the face". This is not to say that ordinary things around us cannot be considered as "scientific" as science indeed studies them. However, not all scientific entities are like ordinary entities. Some of the scientific entities are "elusive and hard-won" since they are discovered and understood only after sustained study (ibid., 2). Moreover, not all ordinary objects or phenomena are of interest for science or become amenable for scientific study (Daston 2000a, 15). That is, only some of the general entities and phenomena become "scientific entities". Few of the characteristics that qualify entities of science specifically are: being regular, "quantifiable, manipulable, beautiful, experimentally replicable, universal, useful, publicly observable, explicable, predictable, culturally significant, or metaphysically fundamental" (ibid., 16). Given this wide variety of characteristics, it is not surprising that the category of scientific entities consists of diverse kinds of entities. For instance, some of the important kinds are (Arabatzis 2011, 379):

1. naturally occurring entities and artificially produced entities (e.g., a novel chemical compound, genetically modified entities). In this spectrum, even natural and artificial processes, like e.g., photosynthesis and laser, can also be included
2. historical things (like biological species) and ahistorical things (like electrons)
3. things that can be observed (like leaves and fire) and things that are observed only through indirect means (e.g., virus and quarks)

Among the different dimensions along which the nature of scientific entities can be analysed, in this chapter I want to focus on two specific aspects: their unobservability

5. Here I am asking a general question: among the variety of things, which of them qualify as "scientific entities"? Thus, I am not making any claims yet about the reality status of either scientific or non-scientific entities. It is important to recognise the spirit of this question because there are some proposals — like naturalism and physicalism — according to which there is nothing like "non-scientific entities". These views argue that our realistic commitments should be limited to the ontology provided by modern science since non-scientific entities can be eventually reduced to scientific entities. I discuss these views in the later section of this chapter.

and their relationship with theory. The reason to consider these two arises primarily because, as I will show subsequently in this chapter and throughout the thesis, these aspects are crucial to the study of photons. With regard to the unobservability aspect of scientific entities, electrons, black holes and other physical entities are quite unlike ordinary things. We have come to become aware of their presence around us in a way that is far different from things like ants and spoons. These things are given to us amidst the activities of science, for instance, while theorizing or conducting experiments. History of science is filled with case-studies about the various modes through which we encounter these entities. Some of the entities like positron have their origin in scientific theories. Contrast to these, presence of certain entities like electron are hypothesized in order to account for the experimental observations.[6] That is, these entities are not given to our perception like the other macroscopic things. Moreover, ordinary things are not dependent on a theory in the same way as scientific entities. We can even say entities like electrons and phlogiston do not just exist, but they subsist on their respective theories. When a theory is superseded, the theoretical entities that inhabit the theory also cease to persist. With this brief remark, in the following sections, I will discuss the nature of scientific entities through the exploration of these two aspects about them.

1.2 Nature of Theoretical Entities

A convenient starting point for exploring the nature of scientific entities is by analysing the intimate relationship shared by a theory and the entities it constitutes. In this section, I will start by discussing few important interpretations about theoretical entities and their dependency on theory. Here, I will specifically focus on Quine's and Maxwell's analyses. After mentioning them, I will bring out few drawbacks about these views in the last subsection.

1.2.1 Theoretical Entities: Ontology of a Theory

Quine's view on theory and objects is a good place to begin the review on theoretical entities. His view has been an influential one for both modern Western metaphysics and philosophy of science. He provides a general notion of theoretical object that is applicable to ordinary, scientific and abstracts objects alike. This summary of his views provides an introduction to various topics related to theoretical entities — the importance of identity and individuality for objects, the notion of physical entity and the stance of structuralism.

6. It should be noted that unobservable entities are not unique to science alone. This kind of entities are ubiquitously present in literary, religious, philosophical and cultural thoughts. For instance, important concepts like god, soul, mind and numbers have been believed to be unobservable.

Problems of references and ontologies

One of the central aim of Quine was to analyse and systematize our ways of talking — about things in general and our scientific theories. His suggestion was that if we regiment our talks through logic and naturalistic commitment, our statements would be simpler and would also reflect the structure of the theories we are committed to. It is in this act of regimentation and paraphrasing of our thoughts, Quine's notion of objects is situated. Therefore, I will briefly highlight this project and describe it to introduce Quine's conception of objects. Even though our common language facilitates the inter-subjective communications, the ordinary way of talking — when properly scrutinized — is quirky and ambiguous. Among the various questions and problems that nature of language throws at us, figuring out what is being referred and how it is being referenced is especially difficult to figure out. Even the simplest case of reference — the one through direct ostension — is filled with uncertainties, as Quine demonstrates in his well known example of "gavagai" (Quine [1960] 2013, 25). In other cases — like talk about non-existent things (like *jatayu*), possible things ("tomorrow's dinner"), mental entities ("My thoughts about this topic") and mathematical entities (like numbers) — it is not possible to pick and point what exactly is the object of reference (Quine 1963, 2-7). By problematising reference, Quine is asking a simple and yet profound question: should we think that all things that occupy the object-position in our sentences are indeed *objects* and do all these objects *exist*? To put this question more starkly, does the "idea" in "I have this idea for tomorrow's party" exist similar to the "key" in "the scooter's key is missing"? A straightforward response to this question could have been made using the argument of direct ostension. However, after Quine's argument against that, there seems to be no easier way to settle any of these questions. The other option is to widen our ontology (like Meinong did) and accept whole heartedly all the things our ordinary way of talking demands. But all is not fine with this ontology - there is no clarity about identity of these things, how many of these things exists, etc. So, redundancy creeps in and it becomes difficult to decide what should be the crop and how to remove the weed.

Quine's solution and approach

These problems quite evidently call for a better understanding of *objecthood*, i.e. what is it to be an object? And it is this that Quine attempts to answer. Given the way he frames the above questions largely in the context of talking about things, he begin by investigating the intimate relationship between language and knowledge. Since the acquisition of one is deeply related to that of the other, he analyses how a child acquires and masters the language over time. This approach, which he calls the *genetic style*, provides an insight into the working of referential claims and their structure (Quine [1960] 2013, 113). Understanding how reference works is just one part of the problem. The referential sentences now have to be rephrased such that this process of referencing is regimented and the objects of reference are clearly laid out. This process of reconstructing the ordinary language sentences can be done in several ways and Quine here prefers using first-order logic (Hylton 2007, 252).[7] This task involves paraphrasing statements, which includes our ordinary object-oriented talks and our scientific theories, using tools of symbolic logic like quantifiers, variables, general terms that act as predicates, and truth-functional operators (ibid., 254).

To illustrate this paraphrasing, consider the simple sentence "any citizen can vote". Here, "citizen" is a *general term* that does not refer to any definite thing specifically like names do; "any", along with other variants like "every", "all", etc., are the *quantifiers* since they quantify the range of things to which the general term is applicable. Therefore, the paraphrasing of this sentence in first-order logic will give: $(\forall x)(Cx \supset Vx)$. This reads as — for all humans (which is represented by "$\forall x$"), if a human is citizen (which is represented by "Cx"), then he/she can vote ("Vx"). An ordinary sentence and its canonical form are not exactly synonymous with each other (Quine [1960] 2013, 145). The logical rephrasing can accommodate further regimentation, like e.g., including the condition of "humans of a country", etc. Nevertheless, in its present form, the canonical representation of the sentence exhibits clearly the actual objects involved. Prior to rephrasing, there were several ambiguities in the ordinary sentence: is "citizen" an object? does "vote" also qualifies as an object? Once put in the logical form, it is clear that "citizen" and "vote" are not exactly referents to objects; instead, these are just predicates that qualify the implicitly present objects — humans. Since humans are the specific *values* for the variable over which the predicates qualify, these are the actual objects. Through this, we can understand Quine's definition of objects: they are the specific values that the variables of a particular canonical sentence can accept (Quine 1982b, 8).

Central role of identity and individuality

Quine thinks, therefore, that the objects referred at the surface of ordinary language sentences are not the ones to which we should be committed. Instead, those objects that resurface after sentences have been subjected to regimentation are the ones that should be taken seriously. The objects found in the logically regimented sentences, unlike their counterparts in ordinary language sentences, have clearer sense of *objecthood* because considering objects as variables brings to the forefront two essential characteristics — criteria of *individuality* and *identity*. To be considered as range of values applicable to a particular general term necessarily implies that there is prescription to evaluate whether a

7. According to Quine, logic not a language; it is just a system of syntax. As Hylton (2007, 252) clarifies, for Quine, "Logic is not exactly a language in the ordinary sense. It has no predicates of its own, except for the identity predicate; so it has no sentences of its own apart from the laws of identity . . . But the syntax of logic becomes a full-fledged language when we add a vocabulary consisting of predicates adequate for the particular matters we are discussing".

given entity is an individual of a particular kind (i.e., evaluating individuality) and a way to differentiate two instances of the same kind (i.e., ascertaining identity) is present. Thus, when we are speaking about a particular kind of object, it is presumed that we know what it is to be an *individual* instance of that kind. Or to frame this point in an another way, to know how to refer to a particular kind of thing is to know what counts as a single instance of that kind. This clarity of what is to be an individual also enables us to distinguish two instances of the same kind. And it is here we see the emergence of object's *identity*. Both individuality and identity are so crucial that, inability to ascertain either of them for an entity suggests that the particular predicate is not able to distinguish different values of its variable and, hence, does not chalk out a clear object. Quine puts this tersely in the form of a dictum: "there is no entity without identity" (Quine 1982a, 102).

Objects and ontologies

According to Quine, our statements has to adhere to logical regimentation along with the demand for clear criterion of individuality and identity. Whenever statements step outside this strict guidelines, the basic aspects of objects used might not be defined properly. Therefore, the canonical notation of quantification makes the sentence's "ontic content explicit" (Quine [1960] 2013, 223). However, since these strictures provide general predicate terms the complete authority to create objects, this also implies that any general predicate that satisfies the required criteria invariably defines a particular set of objects. In this sense, all that we require to have certain kind of object is to suitably define a general term. This realization about the power to create objects raises few concerns. First, the ease of stipulating a specific type of object suggests undesirable multiplication of objects since new predicative terms can be formulated at will. This possibility might come as a surprise since this goes against the very intention with which the regimentation was stipulated in the first place — to restore the order and confusion regarding our ontology. Even though the concern is justified, there is nothing to worry here according to Quine. This way of conceiving objects might give the power to formulate newer general terms, but the criteria of identity and individuality will make sure that the kinds of objects suggested are not vague. This guideline will work even when there are different general terms referring to the same entity. For instance, the predicates "the evening star" and "the morning star" suggest different referents; but both these predicates pick the identical object (Quine 1963, 9).

It is indeed nice to see that Quine's stipulations neatly resolves this apprehension. But it is time we ask Quine an important question about his framework: can any kind of object be allowed in our ontology? It is clear that objects are instances of certain well defined predicates, like e.g., humans, fictional characters, theoretical models, currencies, etc. However, this notion does not guide us whether mental or abstract objects are allowed apart from the well-acquainted ordinary things. It is evident that Quine's conception of objects is plain and does not specify anything about the kinds of objects that are possible. He prefers this compared to a conception of objects that restricts the ontology in specific ways. He mentions that "a finite and listed ontology is no ontology" (Quine 1982b, 7). For instance, if the notion of objects is qualified as only those entities that are spatio-temporal beings, then this condition does not accommodate, by definition, abstract entities in the ontology. For Quine, to stipulate objects as beings of a specific kind or to deny the possibility of certain kinds is to believe in some a priori philosophy. But, this subscription does not sit nicely with his central belief in *naturalism*, which considers science to be the sole arbitrator of knowledge (Hylton 2007, 7; Quine 1982b, 21). Given this stance, Quine's policy is to subscribe to an ontology which is demanded by our current best scientific theories. Therefore, to reiterate Quine's guide to ontology, our theory of objects should not specify what kinds of entities exist; instead, we should ask science to provide the ontology.

Objects of theory and structuralism

This naturalized ontology, however, is vast and comprises of various kinds of objects. On the physical side of the spectrum, there are various kinds of entities like sense-data, ordinary objects, physical objects. Apart from these, there are several kinds of abstract entities like numbers and sets. Therefore, the question raised at the beginning still persists: how to decide what to include in our ontology and what to dispense off with? For this, Quine mentions that there should be a reductionist approach such that the vast number of entities should be reduced to few simpler kinds such that the final ontology is lean and only those entities that are required by our theory of the world should be entertained. With this dictum, Quine suggests nominalisation of all references to sense-data using the notion of *physical objects*. Here, a physical object can be understood as "the aggregate material content of any portion of space-time, however ragged and discontinuous" (Quine 1976, 497). This paraphrasing of sense-data, according to Quine, greatly simplifies our account of the world (Quine [1960] 2013, 2). Similarly, this notion of physical objects is broad enough to accommodate the notion of ordinary objects (Quine 1976, 497). Also, the other motivation for promoting this ontology comes from the fact that scientific theories, to which Quine gives primacy, also need this kind of entities. These theories, apart from physical objects, also require abstract entities like attributes, numbers and relations. Since all abstract entities can be reduced to pure sets, Quine concludes that naturalized ontology consists of physical objects and "grudgingly" includes abstract entities too (Quine [1960] 2013, 218; 1976, 502).

It is interesting to observe here how Quine is making ontological decisions. Placed well within the boundaries of his naturalism stance, these decisions were guided by simplicity and "considerations of systematic efficacy, utility for theory" (Quine [1960] 2013, 218).

Therefore, Quine's basic ontology building recipe is to first consider the scientific theories that are successful; analyse them and figure out the objects these theories demand; among these kinds, attempt simplification and finally distil out the basic constituents of ontology. The crucial and deciding factor about this approach is the starting point — what ontology we end up with completely depends on the theories we start out with. Another set of theories would suggest a completely different ontology. The claim that theory decides ontology implies that there is no theory-independent objects and more importantly, all objects are posits of a particular theory (Hylton 2007, 298). This view gets grounded in Quine's structuralist interpretation of theories, according to which theory is nothing but the structure it proposes. As he states, "structure is what matters to a theory and not the choice of its object" (Quine 1982b, 20). And in another place, he mentions "sentences, in their truth or falsity, are what run deep; ontology is by the way" (Quine 1978, 165). For him, the structural interpretation of theories also explains theory replacement and reduction of one theory to another, both of which are important for science.

Few Comments on Quine's Conception

Quine's *object* is an entity that dwells within the realm of our language. This notion of object is not something that objectively exists out there in the world, independent of us. Of course, this claim is not surprising once we recollect Quine's position about the relationship between language and reality. For Quine, our language and theory completely constitutes our ontology and there is no reality that is outside this. And given that he spends enormous time in regimenting our talk and sharpening our references, the question "what is to be an object?" should get clarified within the language side and not the other way round. That is, according to Quine we cannot pick or refer to objects unless our language provides a way for talking about them. Therefore, "objecthood, and termhood, are theoretical matters" (Hylton 2007, 304). Moreover, given that Quine's object is a language entity, I think labelling this conception as *logical object* neatly captures its central aspect. This is because at the heart of considering objects as *values* of a predicative variable lies the criteria of identity and individuality. If these criteria are not provided, being a "value" does not make sense. Since Quine requires a generalized notion of objects that is applicable to any theory, the logical criteria enables to postulate a bare minimum conception of objects. However, formulation of object using logical criteria makes it ontologically neutral. This aspect of objects being non-committal towards ontology is clarified by Quine in the following quote:

> how are we to adjudicate among rival ontologies? Certainly the answer is not provided by the semantical formula "To be is to be the value of a variable"; this formula serves rather, conversely, in testing the conformity of a given remark or doctrine to a prior ontological standard. We look to bound variables

> in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine...says there is (Quine 1963, 15).

Thus, for Quine, photons would be those objects that are found in a specific theory and are necessary as long as that theory requires them. And, nothing apart from the theory can tell us the nature of these entities.[8]

1.2.2 Theoretical Entities: Constitution by Law-like Principles

Apart from Quine's view, there have been other attempts at interpreting the notion of theoretical entities. Even though these and Quine's view have some overlap, they disagree at important places. Here, I will consider the interpretation provided by G. Maxwell (1962). In the paper titled "The Ontological Status of Theoretical Entities," Maxwell is interested in providing a realistic ground for theoretical entities since they are unobservable. After providing several arguments against the anti-realism of theoretical entities, which I will consider in the subsequent sections, he discusses the "ontological status" of theoretical entities. For Maxwell, "in order to speak at all about any kind of entities. . . to consider their existence or nonexistence, one must first accept 'linguistic framework' which 'introduces the entities' " (ibid., 22). The similarity between Maxwell and Quine is not surprising given that they are drawing from Carnap and other logical positivists' conception of language and reality. In Maxwell's view, a "linguistic framework" consists of (i) logical formation and transformation rules which can in turn generate sentences (ii) a set of confirmation rules (iii) a set of sentences whose truth-values can be easily decided (iv) lastly, a set of law-like sentences that can assign non-ostensive meaning for all non-logical terms present in the framework (ibid., 23).

Among the several "components" of the linguistic framework, it is clear that it are the law-like sentences that bring theoretical entities into "life" by not only defining these entities but also their characteristics. Here, Maxwell is non-committal about whether laws can provide only analytical meanings or they can assign synthetic and empirical meanings as well (ibid., 24). Thus, for Maxwell, as long as a well confirmed theory is there, the accompanying theoretical entities also exist. These entities not having certain characteristics possessed by ordinary entities should not be the reason for denying their existence (ibid., 25). With this, he suggests a piecemeal approach of deciding the nature and status of theoretical entities. For instance, he confesses that it is not straightforward to answer whether electrons qualify as physical objects, even though they have rest mass and

8. Here, one of the reviewer's raised the following concern: how is that "interactions in the world" cannot tell us about the nature of the concerned entities and only theories can? Even though scientific theorising proceeds in the way the reviewer has pointed out, Quine is not sympathetic to that. This attitude of Quine can be seen clearly in his essay "Whither Physical Objects?" 1976. In this essay, he argues that the fundamental ontology one should commit to is that of "pure sets" because all theories in science (whatever physical interaction they are accounting for) can be mathematically reduced to set theory.

causally interact with other entities. Even though, a thoroughgoing "rational reformation" can provide the answer, he points that avoiding conceptual blunders while talking about these entities is more important. In the case of photons, Maxwell offers a response which is equally vague. He mentions that in spite of photons not being physical entities as they do not have rest mass, they are " 'every bit as real' as electrons" (G. Maxwell 1962, 25-26).

1.2.3 Theoretical Entities: Some Concerns

As the above discussions highlight, theoretical entities are often characterised as denizens of a theory. This interpretation has been put into action to exhibit the ontology of various theories. To mention few examples, Brading (2012) analyses the constitution of bodies in Newtonian classical mechanics; entities constituted by Galilean transformation rules have been discussed by Castellani (1998); and the nature of entities found in quantum mechanics have been dealt in Mittelstaedt (1998). This understanding of theoretical entities has also played a pivotal role in the establishment of structuralist interpretations of scientific realism. The connection between these two stances was already touched upon in the case of Quine's framework and more about the modern philosophy of science arguments will be considered in the following sections.

In spite of being the dominant interpretation of theoretical entities, the above understanding has certain drawbacks. One such issue has been articulated by Achinstein (1965). In this paper, even though Achinstein points to the difficulty of demarcating the general category of theoretical *terms*, this argument equally applies to the notion of theoretical *entities* as well, since the general category includes terms referring to entities apart from other things like phenomena (like "interference") and properties (like "resistance"). Achinstein notes that this category of terms are usually demarcated either by defining them as "unobservable" or as "theory-laden". With respect to the latter definition, Achinstein is largely interested in demonstrating that it is not possible to "draw the broader distinction" between theoretical and non-theoretical terms (ibid., 202). Given that there are several ways terms can be dependent on theories, it is not possible to prepare a general list of "theoretical" terms. Therefore, "if categories are invoked to mark the similarities which do not exist they will need to be a good deal more specific than the 'theoretical' and 'non-theoretical' classifications too often presupposed ... " (ibid., 203). In the paper, he is specifically responding to Gilbert Ryle's notion of theoretical terms. According to Ryle, theoretical terms are the ones which carry "the luggage" of one particular theory (ibid., 199). Given this, to know the theory-laden meaning of a term, a specific theory needs to be specified in whose context it could be evaluated whether the term is theory-laden or not (ibid., 200). As a response, Achinstein first shows that there are different kinds of "ladenness". It is not the characteristic feature of theory-laden terms alone to be understood in the context of "system of beliefs or set of facts". Apart from these, there are other kinds of terms that can be understood only by reference to certain "scheme" (Achinstein 1965, 199). Achinstein considers the example of "university-laden terms", "instrument-laden terms" to indicate that "theory-laden terms" are not the only ones which cannot be understood independent of a context. With this general remark, he argues that there are different ways in which terms can be dependent on the principles of a theory (ibid., 200). Some terms depend directly on the laws of a theory; there are terms which appear in the characterisation of a theoretical entity; other terms play an important role in the derivation or application of a law, even though they do not appear in the final articulation of the laws of a theory. Achinstein also adds that there are theoretical terms that are part of more than one scientific theories (ibid., 199). And in all these theories, terms might not be dependent on theory in the same way. By demonstrating the complexity of defining "theoretical terms", Achinstein argues that there is no general category of these terms that can be chalked out.

Apart from this general difficult, there is another important weakness regarding the above discussed conception of theoretical entities. The drawback that I wish to explicate can best be described through an example. During the development of early quantum mechanics, Planck and Einstein differently interpreted the notion of light-quantum, the elements of energy required by the new radiation law. For Planck, light-quantum was nothing more than a theoretical artefact needed to derive the final law. Einstein, on the contrary, held that light-quantum are physical entities which constitute radiation.[9] Therefore, "light-quantum" not only had unique meaning in each of these theories, but these two interpretations disagreed with one another about the physical significance of the term. In this kind of scenario, the theory-constitution view of entities does account for the difference in the meaning of "light-quantum". However, this view does not help in further clarifying which of these two interpretations should be preferred. By concluding that Einstein and Planck conceived the concept differently, this interpretation of theoretical entities ends up stating the obvious fact that meaning of a word is specific to the context. Thus, by confining the understanding of theoretical entities to "linguistic frameworks", this interpretation of entities falls short of being helpful for science, which is interested in making claims that are true of the physical world.

1.3 Nature of Unobservable Entities

Even though a theory constitutes its entities, their observability is equally important. Since a theory should map to a set of natural phenomena, observability of entities involved becomes a crucial factor. The criterion of observability becomes even more important in the context of physics, as the modern physical theories are completely about entities that

9. I will discuss in detail the difference in their interpretations in section 4.2.

cannot be accessed by humans directly. Given this, it is not surprising that the notion of observation is a widely discussed topic and these debates have played a vital role in clarifying the notions like scientific observation, indirect perception, observation of data, etc.[10] In the present section, I intend to discuss the notion of unobservability and how it has been positively understood.

The unobservability of some scientific entities is compensated in various ways. In some cases, the models of entities are hypothesised based on certain theoretical assumptions. The well-known examples here are the models like corpuscles and billiard balls for atoms and particles. Models — e.g., the conception of light based on water waves — are also arrived through analogical reasoning. However, because of this indirect ways of accessing the entities, there has been a strong hesitance about considering these entities as real. This criticism has been basically motivated by the dichotomy between *theory* and *observation* (Achinstein 1965, 198). For the empiricist stance, theoretical entities serve only pragmatic value and there is no point committing oneself to the "reality of supra-empirical entities". In this view, theory along with its entities reduces to "merely an instrument, useful for organizing observation statements and (thus) for reproducing desired results" (G. Maxwell 1962, 5). Theories and their terms perform only the intermediary role in this conception such that theories might be considered as instruments that are essential for constructing and arriving at observational statements which are true of concerned phenomena (ibid., 18).

Indeed, claiming the existence of hypothetical unobservable entities based on the success of certain theoretical claims is a concern. However, denying the category of unobservable entities demands a definition of "observability" and providing one such that it yields neatly separated class of entities is quite challenging. It might appear that these two categories can be easily separated by distinguishing observations that are directly perceived through our senses from those that are mediated to us through instruments and other means. This distinction might appear to satisfy staunch empiricists and logical positivists (Hacking 1983, 169). But, the categorisation of "observation" as that which is non-mediated and direct has several problems. As G. Maxwell (1962, 7) argues, this definition would classify even the common sense scenarios like things perceived through air, spectacles and glass windows as "unobservables". Given this, Maxwell concludes that instead of a clear demarcation, there is only a continuum in which certain things are more mediated than others. In response, it might be pointed out that theoretical entities are those which are "unobservable in principle". Maxwell brushes off this definition by suggesting that this kind of a priori principle of unobservability is futile (ibid., 15). Also, entities which are presently unobservable can be observed in the future and because of this possibility, some philosophers have preferred to label this category as "hidden" entities (Arabatzis 2011, 382n15).

10. See Shapere (1982), Bogen and Woodward (1988), Bogen (2017).

Moreover, as recent discussions on scientific observation have noted, it is important to recognise the limitation of our own perception, like for instance the dependency of human eyes on electromagnetic radiation (see Shapere (1982)), to appreciate instrument perception. And as Maxwell correctly claims, which much later Hacking (1983, 180) also favours, one can be trained to observe theoretical entities (G. Maxwell 1962, 14). This makes possible understanding "observability" operationally: theoretical entities can be reported such that "reasonable sophisticated language user" can evaluate the respective claims (ibid., 13). In the act of reporting, many pragmatic aspects play significant role. For instance, as just noted, knowledge and training of observer play important roles. Apart from that, context of observation is also a factor that needs to be taken into account since what gets observed and reported depends on the specific enquiry (Achinstein 1965, 194). The context of enquiry also decides how "observation" is defined. For instance, in ordinary situations, there might be distinction between observing an object around us and a scientific entity like electron. However, in a cloud-chamber experiment, electrons might be "directly" observed contrast to inferring the presence of neutrino. And in other instances, certain properties like length and temperature are directly measured compared to the indirect measurement of resistance (ibid., 196).

The above refurbished notion of observability helps in accommodating theoretical entities usually found in physics. Adopting the broader notion of observation is also crucial to understand how these entities are held to be "real" in practice. As several case studies highlight, a robust understanding of unobservable entities were arrived at through both theoretical and experimental inputs. However, there is no standard "methodology" of bringing together theory and observation to finally arrive at the object of interest. The interplay between theoretical and observational inputs has been very unique in each case. To illustrate this and to generally highlight how theory and observation crucially guide in analysing unobservable entities, I will briefly consider few case studies.

To begin with, consider the discovery and verification of electron through, as Norton (2000, 72) calls it, "over-determination of constants". According to this method of assuring that theoretical claims are "true", a claim is evaluated across several experiments, each one differing from the other. In these cases, a particular constant of the concerned theory is verified. If the outcome of these experiments tally with one another, then the claim is considered to be true. In the context of the discovery of electrons, Thomson's conclusion that cathode rays are nothing but stream of particles was based on the over-determination of the value of mass-to-charge ratio $\frac{m}{e}$. Thomson examined the deflection of cathode rays in different experimental scenarios. In spite of the change in various experimental parameters — the shape of tube, the pressure of tube's gas, the material of cathodes, etc. — the deflection of rays exhibited the same value of $\frac{m}{e}$ (ibid., 75-76). This over-determination established that cathode rays have to be constituted, not by ether waves, but of corpuscles which get affected by electric field's strength. Through these experimental observations, Thomson was able to arrive at the value of charge and mass of electrons.

In Norton's discussion, another important case study is found that utilises a completely different approach to bring together theory and observation. In this methodology, which Norton names as "demonstrative induction", observations are the starting points and a theory is formulated based on these evidences (Norton 2000, 78). Bohr's 1913 postulation of the constitution of atom can be considered as a case study for illustrating this approach. Bohr starts by theorising the observational results of hydrogen's emission spectrum using Planck's equation and deduces the presence of electrostatic model of electron orbits. In order to further elaborate on the nature of these "orbits", Bohr utilises the observational data of Hydrogen's emission spectrum to infer that these orbits' energy has to be quantified. Through this, Norton shows that "Bohr's theory did not merely depend on its success in saving the phenomena of atomic spectra... Bohr's theory was *inferred* from that phenomena" (ibid., 89,emphasis added).

The above case studies briefly illustrate how theoretical unobservable entities' existence have been argued for and how their characteristics were arrived at. As can be noticed, the proposal of unobservable entities did not emerge only in theoretical context. Experimental observations have not played merely the role of confirming theoretical claims. Instead, observations have helped theoretical analysis jump over hurdles and have guided further understanding of phenomena and entities involved.

1.4 Disinterest about Physical Entities

1.4.1 From Entity-realism to Anti-realism

Right from its conception, physics, with few notable voices of dissent, has generally embraced the presence of unobservable theoretical entities. In this sense, the commitment towards this kind of entities have been an integral component of scientific realism stance. Usually referred to as "entity realism", this position largely believes that entities presupposed by current theories — especially the varied kinds of particles — are actual constituents of the physical world (Psillos 2005, 247). This stance usually gets summarised through the well-known motto provided by Hacking (1983, 23): "if you can spray them, then they are real". As the caption notes, the motivation for this position is grounded in the experimental approach towards doing science. The presumption of entities' existence is not a naive one as these conceptions can resist the "imprecisions of our foresight and understanding" and have the capacity to "surprise us" through the "dissonance" they can bring about between "different ingredients of experimental practice: high-level theory, models of the phenomenon under investigation, our understanding of the apparatus we employ, and the material realization of the experiment" (Arabatzis 2011, 383). Also, these unobservable entities not only ground the unification of several observational phenomena and the respective theories, but also act as anchors during theory-change (Arabatzis 2011, 385). As Hacking mentions, "the theory of positrons might be abandoned or superseded by a totally different theory about positrons, leaving intact what had, by then, become the class of observation sentences represented by 'that's a positron'" (1983, 179). However, as Psillos correctly notes, articulating entity-realism in the above ways, makes it a "selective realist position" (Psillos 2005, 247). Hacking's comment on positron reveals an intrinsic hierarchy of commitments: theory is dispensable, but not it's entity. Psillos considers that this bias of entity-realism makes it an inconsistent position since an entity's reality alone cannot be held as true without drawing some aspects provided by the theory (ibid., 248). With this, Psillos identifies two versions of entity-realism: the "thin" version commits only to the entities that are causal agents of a phenomenon; the "thick" version goes further and attributes causal properties to these entities.

Commitment to the reality of theoretical particles did enjoy support during classical physics and the initial phase of modern physics in the twentieth century. However, due to several reasons, not many contemporary physicists and philosophers are in favour of entity realism. This shift in the status of entities might come as surprise since fundamental entities had been the paradigmatic candidates of reality. As the history of physics indicates, this disinterest in physical entities has been a gradual trend. Among the various factors that has led to this shift, the prominent one is the mathematisation of theories in physics. This style of doing physics brought with it an attitude, as subsection 1.4.2 will highlight, that no more dwells about physicality of entities involved and considers mathematical descriptions as sufficient and appropriate representations of physical reality. The movement towards excessive mathematisation was not without motivation. As I will show in subsection 1.4.3, quantum mechanics posed severe challenges to the classical conception of physical entities. Parallel to these changes, as subsection 1.4.4 elaborates, the overall success of science, especially in physics, deeply influenced modern development of philosophy. In western analytic tradition, there were proposals to naturalise philosophy, according to which philosophical enquiries should be bound within the confines of scientific ontology and epistemology. Naturalisation of philosophy, apart from other reasons, has given rise to the current dominant stance in scientific realism: structuralism. According to this, which is discussed in subsection 1.4.5, realism about scientific theories is commitment towards theories' structure and not to their objects. Thus, the extreme version of this view — ontic-structural realism — is anti-realist about physical entities.

1.4.2 Transition from Material to Physical

Up until the eighteenth century, even though natural philosophy studied various kinds of objects, all of them were material entities. In this sense, there was clear conception of what they are fundamentally. However, this clarity about the nature of the very subjects of study drastically deteriorated from the nineteenth century onwards. This was a gradual outcome and was fuelled by several fundamental changes in the way science was being done, the foremost among which is mathematisation of sciences. Here, it is important to specify which sciences underwent this change as Kuhn (1976, 5) mentions, classical sciences like astronomy, optics, statics (e.g., hydrostatics), harmonics, were mathematical right from the Greek times.[11] The use of mathematics has been so central for these subjects that Kuhn specifies that they can be described to belong to "a single field, mathematics" (ibid., 6). So, the use of mathematics in sciences has a long history. However, in the seventeenth century, new subjects — like electricity, magnetism, sound, heat, light, and others — were brought under the fold of natural philosophy largely due to the influence of Francis Bacon's experimental approach (ibid., 10). It were these "Baconian" sciences that were mathematised in the nineteenth century (Buchwald and Hoon 2003, 170). Here, "mathematisation" of a science stands for the theoretical advancements that were possible only due to the use of "higher mathematics" (Kuhn 1976). The mathematisation of electricity, heat, and others topics finally led to the formation of *physics* as a "professional, unified, quantitative and exact discipline" during the nineteenth century (Buchwald and Hoon 2003, 165).[12]

The mathematisation of sciences and the rise of physics instilled a new attitude towards physical entities: physical interpretation of entities and phenomena that they partake in was no more required for the doing of physics. Even though this shift was recognised early on, there were only few physicists who raised concern about this. To mention an instance, the French natural philosopher Denis Diderot said "the act of generalisation tends to deprive concepts of their sensible aspects. As generalization advances, corporeal spectra vanish; notions move from the imagination to the understanding and ideas become purely intellectual" (Gingras 2001, 403). This anxiety was not uncalled for. By the mid-nineteenth century, the style of theorising had changed drastically and there was progressive attenuation in the habit of providing physical description along with mathematical expressions in presentations of theories. Thus, with the emphasis only on "the syntactical structure of the theory", the "semantic interpretation of its terms" was constrained and this eventually lead to the "undesired and disturbing ontological effect" of "dissolving substances in the acid of mathematics"(ibid., 404).[13]

This transformation of the concept of physical entities has been labelled as *dema-*

11. This argument by Kuhn — to show that the intimate relationship between mathematics and science observed in modern science is also seen in Greek period — suggests that he presumes that modern science has origins in Greek thought. This Eurocentric history of science has been criticised. See Bala (2006).

12. As Buchwald and Hoon (2003, 171) mention, the formation of the discipline of physics was triggered primarily by the Laplacian style of doing science. This way of doing science not only incorporated "algebraic modes of representation", but also adopted new instruments and techniques of measurements, and emphasised on accurate, precise, and numerical representation of experimental results.

13. Gingras (2001, 405) goes as far as claiming that electromagnetic ether, "after a century of unsuccessful efforts at its mathematisation", became discredited because it could not be mathematised.

terialisation. It is not hard to trace back this situation with the very birth of modern science, when Galileo chose to use "geometry the language of physics", and used mathematics to idealise the natural phenomena (McMullin 1985). Nevertheless, apart from the mathematisation of science, there have been other programmatic features of modern physics that have made the traditional elements of reality — matter and material entities — dispensable for physics. One among them is the reductionistic attitude at the heart of physics. As McMullin (1963, 31) notes, "there is a tension between [the] notion of matter and the reductionist mode of explanation... The atoms of today are the reducible structures of tomorrow...". Apart from these aspects, there have been few landmark problems and achievements of modern physics that have also disfavoured the notions of matter and entities. The discovery of mass-energy equivalence at the beginning of the twentieth century is a good example here. By suggesting the equivalence between two properties — mass and energy — this principle led to the "demotion of matter as the sole carrier of the 'reality' label", since according to this principle, massless radiation too is as physically real as material entities (McMullin 2010, 24). "Energeticism", and not materialism, therefore has become an important position in physics and this is reflected through the centrality of energy in the general theory of relativity and other field theories, and also in the dominant theories of cosmology (ibid., 28).

Supplementing to the impact of the energy-mass equivalence principle, the problems faced while conceiving and understanding quantum particles further aggravated the object ontology in modern physics. This deterioration happened in several stages. To begin with, as mentioned above, fundamental entities were no more characterised by categorical properties. To put it starkly, as N. R. Hanson (1963, 554) points, the "shape" and "solidity" of electron, which were central features of entities in classical times, are no more clear in modern theories. Not only that the categorical properties were not available to these entities, the actual properties provided by quantum mechanics — as I will discuss in the next subsection — were observation dependent and probabilistic in nature. To this gradual advancement of dematerialisation, as Shrader-Frechette (1980) points, the revision to the criterion of physical reality has further contributed. Using the developments in particle physics as example, she argues that modern physics considers the reality of particles based only on "the result of detailed study of its properties and its relations to other phenomena, rather than the result of some critical experiment that demonstrates its existence directly" (ibid., 306). A prime instance here is the claim about the existence of quarks that are "in-principle-unobservable" (ibid., 304). This change in the criteria of reality, which are based solely on theoretical implications and not on the empirical grounds, has led to "radical, step toward the dematerialization of the concept of matter" as there has been a shift "from the notion that ... observation is *in-principle inexact and probabilistic*, to the notion that observation is not merely inexact and probabilistic, but *impossible*" (ibid., 307, emphasis added). Adding to these challenges, wave-particle duality, the uncertainty principle and introduction of virtual particles “did even more violence to everyday convictions” of matter and objects (McMullin 2010, 25-27).

With these developments, the concepts of matter and objects are no more of primary interest in science. As McMullin states, “ ‘matter’ in most of its standard uses is coming to be recognised as part of the meta-language of science, not a part of the working vocabulary or object-language of the scientist. It is a meta-theoretic term, one which serves to illuminate many facets of the scientific effort in the ways we have noted above, as well as allowing us to talk about the work of science in everyday language. But in its meta-theoretic senses, it has *no direct function* in scientific theory” (1963, 38, emphasis added). This is largely due to the development of a particular way of doing science which emphasises the use of mathematics. This “shut up and calculate” style does not encourage enquiry about the nature of physicality or about the nature of entities since it presumes “our external physical reality is assumed to be purely mathematical” (Tegmark 2007).

1.4.3 Problems Concerning Quantum Entities

The challenges faced while interpreting quantum mechanics as a theory of entities can be explained by establishing how it deviates from the classical understanding. The properties possessed by quantum entities[14] can be categorised into two groups. These entities do have certain *fundamental* properties — like charge, mass and spin — that make them what they are. Since every kind of particles has unique values for this set of properties, these entities have been qualified as “nomological” entities (Toraldo di Francia 1998, 26). Apart from these properties, quantum particles also have *dynamic* properties. These properties have been labelled as “circumstantial” by some physicists since they depend on the preparation of the system, its evolution across time or the act of measurement (Omnès 1994, 61).

With these two types of properties, quantum entities and their state could be described. The state of a quantum system is usually represented by a vector in Hilbert space, a mathematical abstract vector space. Unlike the classical state representation, a vector in Hilbert space does not necessarily represents a unique distinct state (Shankar 1994, 118). Regarding state observables, in quantum physics there are Hermitian operators that operate on state vectors to get the respective values of the observables. Therefore, for every observable (like energy, momentum, and position), there is a specific Hermitian operator which needs to be used (Hughes 1989, 59). When the quantum state-vector is provided, depending on the observable required, suitable Hermitian operator is used to represent the state in terms of the eigenvectors of the operator. For instance, to measure the spin of an electron, the spin operator, which for electrons has two eigenvectors, needs to be used. This operation provides eigenvalues associated with the eigenvectors for a

14. Even though quantum mechanics can be applied to any kind of entities, the default subjects of this theory are fundamental particles like fermions and bosons.

specific state. Each of these eigenvalues represents the probability of the electron having the respective eigenvector spin-state (Hughes 1989, 64). Since the probabilities of the electron being in various states can be calculated, this procedure is supposed to give us the *predictive state* of the system (Omnès 1994, 117). It is "predictive" because it foretells in probabilistic terms what state the electron would be if the actual measurement is done. Contrast to this, an electron can be "prepared" so as to have a specific state value. This state of the electron can be considered as the *historical state*.

Apart from the probabilistic characterisation, there is another important aspect of quantum states which makes them categorically non-classical. Consider again the scenario of predicting the spin value of an electron. Given that the spin can be in either of the two states, the application of the spin operator on the electron's state vector will assign probability 1 for one of the states and probability 0 for the other. This kind of quantum state where the value is described by one of the eigenstates is labelled as *pure state*. This is of significance because it is only in this kind of state, the value of the prediction for a specific observable will have definite outcome. Quantum systems, however, can be in a *superposition* of all its possible pure states. In this superposed state, prediction does not reveal the specific state of the electron. When expressed through eigenvectors, the corresponding eigenvalues, both of which are non-zero, represents the probability of the electron having that respective spin-state (Shankar 1994, 118). And when the actual measurement is carried out, the electron will be found to be in one of the spin-states depending on the probability distribution. In this superposed state, quantum mechanics provides a non-classical interpretation of the state variables. To highlight the central aspect by which this differs from a classical system, let me contrast these two. In a classical system, the state of the system can be understood as a two valued function consisting of an observable (like momentum) and the specific value of it. In this form, when a particular pair of values are given to this state function (say momentum having specific value), it can give a definite answer whether the system is in this state or not. However, when quantum state is construed as a two valued function as above, this quantum function cannot always provide a definitive answer for a query regarding a particular observable value. If the system is in the superposed state, then the function provides a value in between these two extremities (Hughes 1989, 156).

These non-classical features of quantum mechanics raises concerns about continuing our allegiance towards particle ontology. Because, if we consider quantum mechanics as a theory about individual particles, then certain difficulties are encountered in interpreting the reality this theory depicts. According to Teller, "informal realism about particles" entails that there are individual entities that can be identified across time and there is a way of ascertaining the specific state of these particles (Teller 1998, 126). However, quantum mechanics seems not to respect this dictum given that it describes reality through superposition states. Moreover, the theory does not explain why a system cannot be in one of the pure states that constitute the superposition state (Teller 1998, 126). This inability is a “small embarrassment for quantum mechanics” because what is allowed by the theory, through the superposition principle, is the superposition of these pure states. Now, if the pure states are not possible, how is that the superposition of these states are possible (ibid., 127)?

Because of the difficulties raised by quantum mechanics, several interpretative attempts have been proposed to understand the nature of superposed states. Each of these interpretations differ in the physical understanding of this state and its ontological implications. Among these, three historically prominent interpretations have been the following (Redhead 1987, 45). According to *statistical interpretation* of quantum mechanics, in this superposition state, an observable would have a specific value. It is just that the theory at hand needs to be improved in order to capture this unknown value. In contrast to the above one, the observable value can be understood as being “unsharp” and “fuzzy”. According to this *propensity or potentialities interpretation*, quantum theory is correct and quantum states possess imprecise, probabilistic values. The other possible interpretation of superposition state is the *complementarity view*, according to which the observable value is undefined and meaningless until the measurement is carried out. Among these views, the statistical interpretation provides an epistemological interpretation of the principle. The propensity interpretation has ontological implications. Contrast to these two views, complementary view provides an operational interpretation of the principle.

Among these, the last two interpretations are non-classical in nature. Each of these reject one or the other fundamental classical presumptions about reality. The first interpretation is the only one to interpret quantum physics through the classical reality lens. There have been various attempts of understanding quantum mechanics within the classical framework. For Einstein and few others, since quantum mechanics provide probabilistic distributions, the description provided by it has to be incomplete and this theory needs to be revised or supplemented with new properties (Hughes 1989, 159). Extending from this, *statistical interpretation* approach considers that quantum mechanics always provides values that is true of an ensemble of entities. Only when this value is interpreted for an individual particle, we get probabilistic description (ibid., 163). This interpretation considers that quantum mechanics adheres to classical physics principles. Hughes (1989, 163) lists four central tenets that this interpretation presumes: (i) *Precise value principle* assures that, in a particular state, the system has a precise value for every observable. (ii) *Relative frequency principle* dictates that quantum mechanics provides a relative frequency of finding particles in an ensemble having a specific value. (iii) *Faithful measurement principle* states that measurement reveals the actual value of the observable that the system possessed *prior* to the measurement. (iv) *Local realism principle* presumes that correlated quantum entities after separation can be considered as independent systems (ibid., 172).

In spite of several attempts to provide a full-fledged statistical interpretation suitable for quantum mechanics, there have been numerous criticisms. These arguments have focused on one of the tenets listed above and have argued how that principle does not apply for quantum mechanics. I will consider few of these to show the failure of classical interpretation of quantum mechanics. Among the tenets listed above, one of the important ones is the presumption that observables have specific values. The principle of precise value implies that there are specific values for observables even when entities are in superposition states. It is this premise that makes it possible to assign unique values to each of the observable. This assignment of values to observables can be carried out such that if the observables are related to each other, then by knowing the value of one, the value of other observables can be fixed. But, as Kochen and Specker showed, for a system described in Hilbert space, this method of assigning values to observables is not possible (Redhead 1987, 121; Hughes 1989, 164). To understand the importance of this argument, it needs to be made clear what exactly is being denied here. The presumption of the statistical interpretation was that a quantum state is constituted of several circumstantial properties and all of these properties have specific values. Among these observables, there are few which are related in a way that knowing the value of one observable, the value of others can be corroborated. Kochen and Specker deny the possibility of doing this. As they demonstrate, it is not possible in Hilbert space having more than two dimensions to assign the values of all such related observables. Therefore, in quantum mechanics based on Hilbert space, knowing the value of one observable does not necessarily imply that the values of other observables can be extrapolated.

Another tenet that has been shown to be problematic in the context of quantum mechanics is the local-realism principle. Bell and Wigner have shown that an ensemble based on the principle of local realism should exhibit certain statistical inequality relationship. But, quantum ensemble fails to comply with this inequality (Hughes 1989, 171). Interestingly, the failure to exhibit this inequality indicates the possibility of quantum entities exhibiting non-local interactions. Along with this, there have been several no-go theorems against the possibility of there being hidden variables in this statistical theory. Here, the "hidden variables" refer to those properties that are not directly accessible for measurement but are relevant since they determine the values of measurable properties. The difficulty of providing statistical interpretation for quantum mechanics leads to perplexing situation. On one hand there is a theory which successfully explains the behaviour of quantum entities. At the same time, this theory does not seem to respect its entities' fundamental needs of having a well defined state, fixed value for characteristics, etc. Thus, we end up with a well working theory that does not seem to be about entities at all.

1.4.4 Naturalisation of Philosophy

The current dominant stances in modern western analytic philosophy too disfavour the study of physical entities. To understand this attitude, it is crucial to know the historical development of these stances. As mentioned above, it was only in the nineteenth century that physics and various related branches of science were distinctly recognised as disciplines studying the natural world. Until that time, most of these researches were carried out under the banner of natural philosophy. Therefore, the schism between philosophy and science, which had started few centuries back, reached its completion in the nineteenth century. This rise and success of science invariably altered the nature of modern western philosophy.[15] With these changes, it was not clear how philosophical questions pertaining to the external reality — both metaphysical and epistemological — were to be answered. Should philosophy no more raise these questions since the task has been handed over to the new disciplines that are conceived solely for that purpose? According to this view, philosophy is no more at the helm about this kind of enquiry and should instead receive the knowledge produced by the sciences. In contrast, the other extreme view is the sceptical position where philosophy is the final arbitrator about the possibility of science and its success.

The relation between philosophy and science is still an open question at present and there are numerous views between the two extreme positions presented above. Among them, the dominant trend in western analytic tradition has been to acknowledge the success of modern science and consider that epistemological and metaphysical enquiries in philosophy should be informed by science. This position, which is labelled as *naturalism*, arose during the first half of the twentieth century with the interest of making philosophy more closely aligned with science (Papineau 2016). This position has been articulated and implemented largely in two different, yet related, ways: (i) methodological naturalism, where the method of doing philosophy is guided by science and is made similar to that of scientific method (ii) ontological naturalism, according to which the ontology provided by our current best science should be considered as the starting point for doing philosophy (ibid.).

Methodological naturalism suggests that philosophical enquiries — metaphysical, epistemological, ontological — should be aligned with scientific pursuits and this has been realised in numerous ways. To illustrate an instance of "naturalised" metaphysics, I will briefly discuss a recent view proposed by Ladyman et al. (2007). According to this view, traditional metaphysics is not capable any more to investigate queries about physical reality. Not only modern western analytic metaphysics "domesticates" scientific knowledge naively, its methodology, which is based on a priori reasoning, is incorrect

15. In fact, as historians of philosophy have shown, the schism of philosophy in the west into analytic and continental schools was mainly due to different attempts to carry out "scientific philosophy" (Richardson 1997).

to grasp the external objective reality (Ladyman et al. 2007, 7-27). Given that science is better equipped to discover the natural world, metaphysics should take up the job of "critically elucidating consilience networks across the sciences" (ibid., 28). In this framework, metaphysics is supposed to provide the "service" of "showing how two or more specific scientific hypotheses, at least one of which is drawn from fundamental physics, jointly explain more than the sum of what is explained by the two hypotheses taken separately..." (ibid., 30). When metaphysics is carried out in the above way, it is said to be bound by the *principle of naturalistic closure*. Similar to metaphysics, there have been proposals to bind epistemological endeavours too within the scientific purview. In this regard, Quine's view about naturalised epistemology has been an influential one. Quine, unlike logical positivists, did not consider that philosophy to be a high-order activity compared to other knowledge gaining activities like science. According to Quine, there is no difference between knowledge acquired through philosophy, science and common sense, and in fact, held that philosophy should aim for scientific standards (Hylton 2007, 8). Since for him sensory stimulations are the foundational sources for all our knowledge, he considered epistemological queries are better answered by cognitive science and in this sense, epistemology is a natural enterprise (Quine 1969, 83; Hylton 2007, 15). All these philosophical projects are grounded on the basic premise of ontological naturalism, according to which, ontology provided by science should be the preferred one for philosophy too. In this stance, one of the strongest view is that of *physicalism*, where it is held that the external reality is physical in nature. This interpretation of reality has been interpreted in various ways. Some of these differ in the definition of "physical" and others provide various means through which the socio-cultural reality, biological reality and others could be fundamentally reduced to physical reality (Stoljar 2010, 2017).

1.4.5 Rise of Structuralism

The above naturalistic style had its strongest influence on the sub-discipline of philosophy which studies the nature of science. Since philosophy of science is interested largely about the reality of scientific claims, the current dominant stance regarding this in western analytic tradition is *structuralism*. This view about science has been interpreted in various ways and at various levels. Most of these interpretations are motivated by few important concerns about science: problems of theory change, natures of scientific objects, relation between scientific theory and physical reality, etc. Among these, consider the problem posed by theory change. New theories superseding old ones raises the scepticism about the reality claims of the new theories. Often labelled as "pessimistic meta-induction", this criticism points that the present best scientific theory would eventually be proven false. Thus, "by a simple (meta-)induction on scientific theories, our current successful theories are likely to be false (or, at any rate, are more likely to be false than true), and many or most of the theoretical terms featuring in them will turn out to be non-referential" (Psillos 2005, 96). One of the ways this criticism against scientific realism has been resolved is through structuralism, according to which the realistic commitment towards science should extend only to theories' structure and not to their specific contents, which include theories' entities (Ladyman 2016).

Another motivation for structuralism comes from the problems faced in the context of quantum particles. Closely related to the interpretative problem of superposition states and meaning of measurement in quantum mechanics discussed above, there was another development that further deteriorated the popularity of quantum particles. The development of quantum statistics — Bose-Einstein statistics for bosons and Fermi-Dirac statistics for fermions — showed that these particles do not even possess identity. To illustrate this problem, consider a two particle system such that each particle can be in either of the two states assigned to the system. In this case, classical statistics says that the system can be in any of the four possible permutations. Quantum statistics differs from this and prescribes only three possibilities as it does not differentiate between the two scenarios where each of the particle is in either of the two possible states (French and Krause 2006, 142). As discussed in (1.2.1), the lack of assigning identity and the inability of differentiating entities has serious repercussions on assigning objecthood to them. As Teller (1998, 128) notes, it appears that both the formalisms — quantum mechanics as a theory of individual entities and of non-individual entities — seem to adequately account for the empirical observations. Given this, French (1998, 95) describes that these particles are "metaphysically underdetermined" as the "formalism can be taken to support two very different metaphysical packages". This perplexing situation has given rise to several interpretations about quantum mechanics and its ontology.

Teller (1998) summarises an interesting response by Bas van Fraassen to this situation. van Fraassen is of the opinion that it does not make any sense to say one interpretation is better than the other as these two are equivalent descriptions of the theory. van Fraassen uses an example from geometry to highlight his proposal. Geometry can be considered as a theory about points and relations among these points. In this conception then, other entities like spheres could be articulated using set of points. Equivalently, geometry can be understood as a theory about spheres and relations between them. In this interpretation, points are set of spheres. Even though both these conceptions have "rival world-pictures", each of these can suitably articulate the other conception's propositions. By showing this, van Fraassen claims that even though it is possible that one can commit to a particular interpretation and ontology of geometry, it is wrong to say that geometry prefers one ontology against the other (ibid., 130).[16] In the context of quantum mechanics, van Fraassen thinks that different interpretations — like label-formalism and fork-space-

16. This stance of van Fraassen, which is generalization of Poincaré's conventionalism, is parallel to the duality articulated between lines and points in projective geometry. This will be discussed in 3.3.

representation — are equivalent such that all propositions in one interpretation can be restated or paraphrased entirely through the terms of another interpretation (Teller 1998, 129). Therefore, according to him, there is "demonstrated equivalence of the particle and the particle-less picture" (ibid., 129). van Fraassen articulates this stance in its generalized form, which he calls as *semantic universalism.* A proposition, according to this view, is "general" if "no change of truth value results from a permutation of individuals" (ibid., 131). Teller criticises the above view of van Fraassen. For Teller, one formulation has "a conceptual framework that facilitates saying things that cannot be said with the resources" of the other formulation (ibid., 131). In the context of semantic universalism, Teller argues that if individuals are postulated with haecceities,[17] then the model formulated will not be equivalent to the one with occupation numbers since the model with haecceities will have "more significant propositions" that are not present in the other model (ibid., 133).

van Fraassen's interpretation has some overlap with Quine's view discussed in section 1.2.1. Quine too proposes that a theory can have multiple ontologies and goes as far as to say that ontology is "by the way". This similarity should indicate the proximity of van Fraassen's view to the extreme position of structuralism that Quine adopts eventually. In fact, among the several outcomes of the ambivalence in quantum ontology, *structural realism* has been the one that has become prominent at present. Various versions of structuralism with different ontological "strengths" have been articulated. The *ontic-structural realism* commits to the ontological reality of structures. In the case of quantum ontology, the difficulties of interpreting quantum entities provided an ideal motivation for ontic-structural realists to move away from particle ontology altogether (Ladyman et al. 2007; French 2010, 2014). Since this stance proposes an ontology similar to that of mathematics, certain philosophers have moved away from this extremism and have proposed *epistemic-structural realism* (Ladyman 2016). This position too has not been favourable towards the object ontology as it considers, given its Kantian heritage, that we do not have access to the actual physical ontology. Against these views, there have been few arguments in favour of object ontology (see Chakravartty (2003)). However, with the larger trend towards field ontology, there has been a gradual decline towards the need for arguing for unobservable particles. Thus, the dominant positions in western analytical philosophy of science consider that physical entities have "withered" away in modern science (see Quine (1976) and French (1998)).

1.5 Plan of the Thesis

The above section recounts the current dominant state of affairs in science and western analytic philosophy of science, especially regarding physical entities. In the physics's front,

17. Teller defines "haecceity" as a primitive thisness which makes possible to apply labels, provides strict identity and enables counterfactual switching.

physical entities are understood only along the dimension that favours the modern style of theorising. There is impatience and a general disinterest to explore other aspects of these entities. Modern analytic philosophy, being naturalised, adorns an anti-realistic view towards physical entities. Because of these reasons, there is not much encouragement to study the nature of physical entities at present. This attitude about the relation between philosophy and science does disservice to both the disciplines. The dominant structural interpretations of philosophy largely draws from the theoretical activity of one sub-discipline of science — physics. This view of scientific reality overlooks other aspects of scientific practice, like for instance, historicity of theories and entities, role of experiments, etc. Moreover, the disinterest in physical entities is antithetical to the current practice of physics where several research activities still revolve around entities. These contextually situated, focused investigations of physicists usually do not become topics of interest in philosophy of science since the dominant views, under the influence of the naturalistic stance, subscribe to the policy of keeping science and philosophy separate so that philosophy does not intervene with the actual doing of science. With this, metaphysical and epistemological questions are rendered non-useful for the actual practice of science.

In the thesis, I respond to the above dominant stance about physical entities and also to the philosophy of science that harbours such a view. Instead of engaging with general claims, like motivating entity realism or showing shortcomings of structuralist interpretations, I will focus on a case-study pertaining to a specific physical entity to demonstrate how modern physics and philosophy can work together. This style of analysis moves away from the dominant view which limits the study of entities only to their theoretical contexts. I start the analysis from a specific experimental scenario and show how a critical evaluation of the experimental descriptions can bring out the fundamental presumptions about the concerned phenomenon. By showing the possibility of gaining these insights, I finally demonstrate — much against the presumption of naturalistic stance — the relevance of philosophical analysis for the actual practice of science. This stance about the way of doing philosophy of science resembles views held by other philosophers, like for instance Hacking, Cartwright and Arabatzis, which were discussed above. This stance has also been influenced greatly by Sundar Sarukkai's views about doing philosophy. His interdisciplinary approach is diametrically opposed to the naturalistic stance which puts philosophy and science in different silos. In this approach, which he has demonstrated in his own works and has been discussed over several personal conversations, the primary agenda is to resolve the concerned problem and to this end, tools from any discipline can be utilised. This attitude, right at its foundation, disagrees with the naturalised philosophy by not presuming that a given question belongs solely to the domain of a specific discipline.

After highlighting the broad characteristics about the stance I have adopted for the thesis, I will now discuss the details of the research carried out in the current thesis. Among the various entities that modern physics deals with, I have chosen photon as the object of interest. The motivation for selecting this entity comes from the topic of study that I am interested in this thesis: conception of physical entities. At the beginning of modern physics in the early twentieth century, wave-particle duality became one of the dominant topics that challenged the classical paradigm and shaped the enquiries about physical entities in important ways. This phenomenon or problem — depending on how it is interpreted — was observed in the case of both light and matter. Even though duality of matter came out as a "severe shock to common-sense intuitions" (see McMullin (2010, 26)), duality of radiation has a much longer and richer history. In the backdrop of the classical debate between undulatory theorists (who interpreted light as waves in a medium) and corpuscular theorists (for whom, light is a stream of particles), the twentieth century discovery of radiation having both the characteristics is a landmark turning point. Therefore, duality of light provides an ideal case-study for not only studying the various challenges and attempts made in the present time, but also enable us to see how these interpretations differ from the ones provided during the early modern era. The importance of this topic can also be judged by the amount of literature — scientific, philosophical and historical — that has been produced regarding duality of matter and light. In this vastly studied topic, however, duality of photons has largely been presumed to be yet another instance of this well-understood topic and has not received any focused analysis. Because of these reasons, I have chosen to study the wave-particle duality of photons with the intention of showing how rigourous analysis of this claim can bring better understanding of photons. With respect to this topic, I have engaged with the following enquiries:

(i) The historical and conceptual analysis of wave-particle duality of radiation
(ii) The evaluation of the wave-particle duality of photons
(iii) The implication of the above analysis for the present understanding of photons' behaviour.

In the remaining part of the section, I will summarise these queries by providing the outline of the thesis.

For the concerned research topic, understanding the notion of wave-particle duality is quintessential and the analysis of enquiry (i) is carried out across the following three chapters. In chapter 2, I begin by providing a brief historical overview of numerous interpretations of light, from the ancient and medieval periods to the recent theories about radiation found in modern physics. Subsequent to this, I focus on photons and introduce various interpretations of photons, not only the different theoretical analyses but also the operational understanding of photons. In chapter 3, I explore the notion of duality prevalent in disciplines like mathematics, logic, and sciences and bring to forefront the varied connotations associated with this concept. Specifically, I discuss the notion of duality found in Boolean logic, duality of lines and points in projective geometry, dual spaces in functional analysis, duality in group theory and category theory, dual theories in physics and a brief survey of duality in other branches of science. This broad survey provides the context for analysing the common features among these distinct meanings of duality. In order to do this, I compare the notion of duality with symmetry and highlight the similarities and differences between them. In chapter 4, I focus on the topic of interest: wave-particle duality of light. The main aim of this discussion is to highlight different interpretations of wave-particle duality and emphasise how duality has been one of the important enquiries that has brought forth fundamental insights about radiation. To begin with, I first establish the plurality of wave-particle duality by surveying variety of historical papers and also contemporary physics textbooks. By noting that there is no single interpretation of wave-particle duality, I study three important interpretations — of Einstein's, Bohr's and Heisenberg's — and show how "wave", "particle" and "duality" in these are conceptually different.

In the discussion carried out across these three chapters, there are two novel research outcomes. The first contribution is the survey of different connotations of duality across few sub-disciplines of science, mathematics and logic, and bringing out its salient features. This is an important step towards understanding why the concept of duality, similar to other concepts like symmetry, are so useful for the practice of science. The second novel contribution is establishing the plurality about wave-particle duality in contemporary physics. While discussing this, I also provide arguments for why this plurality was overlooked. As I mention, recognising the differences among the views associated with wave-particle duality is crucial for modern research as the various confusions regarding the current status about this concept arise primarily because the plurality of duality has not been recognised.

After analysing the general notion of duality and specifically wave-particle duality, I focus on (ii) enquiry: the phenomenon of duality in the case of photons. This analysis will be carried out in chapter 5. Here, I focus on the duality claim articulated in the context of two experiments: the anti-correlation experiment and photons' behaviour in the Mach-Zehnder interferometer experiment. These experimental demonstrations put together were considered as a conclusive evidence of single photons exhibiting wave-particle duality. However, I will argue that this claim cannot be substantiated either through the complementarity principle or by claiming that photons behave analogous to classical waves and particles in these experiments. Regarding the claim based on the complementarity principle, there are already several arguments showing the problems of inferring duality through complementarity. With respect to the claim based on analogies, I analyse both particle and wave analogies in their respective experimental scenarios and argue that these analogies are either wrong or incomplete. Thus, the duality claim about single photons loses its strength.

There are several novel aspects about the arguments presented in this chapter. Until now, the duality claims about photons have been considered as just another variant of the usual duality claims found in physics' literature. Contrary to this presumption, in this chapter, I analyse the various duality claims about photons found in scientific literature and identify two variants: one based on the complementarity principle and the other based solely on analogies. As I will show in the discussion, recognising the two different varieties is crucial for evaluating the duality claims. Among these two claims, the claim based solely on analogies of photons has gone unexamined till now. Here, I not only analyse and show the weakness of wave and particle analogies of photons, but through historical and conceptual examination of the experiments, I suggest positive analogies that can be drawn in these scenarios.

The above discussion shows how application of a concept (wave-particle duality) to a new scenario (two experiments of single-photons) without examination can lead to invalid claims. In the subsequent chapter, I explore yet another confusion pertaining to photons. In order to show the non-applicability of wave analogy for single-photons in chapter 5, I highlight the difference between classical optics' and quantum mechanics' interpretations of interference. The stark difference in the understanding of the phenomenon gives rise to an important question: why is that in both these cases, the phenomenon is labelled as "interference"? This question would not have been significant if these two — classical-optics and quantum-mechanics — were the only available interpretations of the phenomenon as this could have been understood as persistence of concepts across theory-change. However, post 1960, landmark changes happen. First, there is an immense advancement in optical technology with the production of maser and laser. Along with this change, new theoretical interventions led to the advent of quantum optics, which revolutionised various concepts of classical-optics. These changes made possible for new kinds of experiments like interference between two independent single-photons. This new kind experiment resulted in confusion about the meaning of "interference" because until that point, this phenomenon was largely understood through Dirac's dictum: "each photon interferes only with itself; interference between two different photons never occurs". The observation of interference between two photons in new experimental setups conflicted with the second part of Dirac's dictum. This confusion is an ongoing debate with no clear resolution yet. Surprisingly, this situation has not been identified as a controversy and no attempt has gone into understanding the source of this confusion. Therefore, in chapter 6, I first examine the interpretations of interference in classical-optics, in quantum mechanics and in quantum optics, and bring out the differences among them. Subsequent to this, I substantiate the controversy surrounding interference. With this established, I analyse the divergent views and identify two important reasons for this controversy. First, I show that physicists misinterpreted Dirac's dictum by not paying attention to the context in which it was formulated. Apart from this, I also highlight how each of these views hold different set of criteria to qualify a phenomenon as "interference".

Given that the close study of the duality of photons brought out the possible misunderstanding about "interference" of photons, the enquiry carried out in chapter 6 pertains to (iii) enquiry mentioned above. There are several novel research outcomes about the discussion carried out in this chapter. Since the controversy regarding interference has not yet been recognised properly, the discussion presented in the chapter attempts to bring it to the attention of physicists and historians. Moreover, the analysis of the controversy presents a comparative analysis of three different views of interference. Finally, by identifying the sources of the controversy, the discussion positively contributes towards the clarification of the confusion. Thus, by showing the invalidity of duality claim about single-photons and bringing to the forefront a not yet well-identified controversy, the main aim of the approach adopted in this thesis has been successfully achieved: the philosophical analysis of two scientific claims provided clarifications that are relevant for the scientific practice. In this sense, the analyses carried out in this thesis exhibit how philosophy and science can work together.

Chapter 2

Nature of Light and Photons

Light has always been central for humans. Not only because our primary way of perceiving the world depends on it, but light has played an integral role in every culture and civilisation. Given this, light has been understood and described through all forms of human endeavours, ranging from philosophy to poetry. In this chapter, I will discuss the brief history of our concepts about light explored along one particular dimension. Various natural and physical views about light across different worldviews are presented. To begin with the ancient knowledge systems, I will consider how Indian and Greek philosophical schools thought about light. After briefly touching upon few medieval ideas, I will move to the pre-modern period. Since there is vast historical literature on all the dominant western natural philosophers of this time, I have taken a pragmatic strategy while surveying this period. Since the primary scope of my thesis pertains to the meaning of "wave" and "particle" in the context of radiation, I have handpicked the important aspects of this dense discussion that are relevant to this topic. And after that, the modern developments of field theory and quantum optics are summarised to set the context for the discussions that will come up in the subsequent chapters.

2.1 The Passage of Light through Ages

2.1.1 Ancient and Medieval Knowledge Systems

Views of Ancient Indian Philosophical Schools

All ancient Indian philosophical traditions have interpreted the characteristics of light and nature of optical phenomena within their unique philosophical frameworks. Among these varied interpretations, it useful to touch upon how Nyāya-Vaiśeṣika (henceforth NV) school, given their realist stance, have talked about light. Vaiśeṣika system recognises *tejas* as one of the nine substances in its ontology. Within this ontology, *tejas* is considered as one of the five *bhūtāni*, the ultimate material elements that constitute rest of the things

(Bhaduri 1946, 50). *Tejas* is translated both as "light" and "fire" since NV considered these to be "different forms" of the same substance (Sarkar 1918, 36). Thus, *tejas* possesses both the characteristics of colour and touch, either of which can be manifested or unmanifested. For instance, colour and touch properties of *tejas* coming out of eyes are unmanifested; in contrast, *tejas* in the context of hot water has its touch property manifested, but colour remains unmanifested (Subbarayappa 1971, 480). This realistic interpretation of *tejas* made NV worldview different from that of other Indian schools. This is because each school's conception of light was dependent on their epistemological theories of perception. For instance, Buddhist schools do not differentiate object from its perceived qualities since object is always perceived possessing them. However, in the case of NV, given that they differentiate perception of light from perception of an object, they argue for the distinction between object and its qualities (Bhaduri 1946, 31). Another instance is how Mīmāṃsaka school differ from NV about the interpretation of darkness. Unlike Mīmāṃsaka school, which considered that darkness is a substance by itself, NV considered this to be merely absence of light (ibid., 36). The realist interpretation of *tejas* grounds several explanations that NV provide. The phenomenon of perception is explained by recognising that visual sense is constituted by *tejas*, i.e. light or fire substance (ibid., 154). NV held that visual perception is enabled by *tejas*: light emitted from eyes, travels and comes in contact with object of perception. Through the motion of *tejas* and its behaviour, various natural phenomena were accounted by Naiyāyikas. Udyotakara, in his work Nyāyavārttika, studies optical phenomena like reflection and refraction (ibid., 65). The fundamental atomistic tenet held by NV is further utilised to explain the behaviour of *tejas*. Transparency of glass is interpreted as the atomistic structure of glass allowing for the uninhibited transmission of *tejas* atoms (ibid., 97). Transformation of objects (when heat is applied) and chemical effects of *tejas* were analysed by philosophers like Shamkara Mishra and Jayantha (Sarkar 1918, 36; Seal 1915, 115-16). These processes were understood as *tejas* atoms coming in contact with objects' atoms to bring about the transformation (Subbarayappa 1971, 748).

Views of Ancient Greek Philosophical Schools

The early Greek philosophical schools have discussed the nature of light in the context of vision. Since light plays a crucial role in the analysis of vision, the views about light held by these schools can only be understood within the larger framework of theories of vision held during this time. The prominent Greek theories of vision can be categorised into two groups — intromission and extramission theories (Bala 2006, 87). According to intromission theories, the process of vision involved things from outside entering into eyes. In contrast to these, extramission theories considered that vision was possible because specific things emanated from eyes went out. Two different interpretations of vision are identified under intromission theories. According to *atomists*, perceiving something involves "effigies" of that thing entering eyes (Darrigol 2012, 5). Light for atomists is constituted by subtler atoms emanated from luminaries, like sun, which make the gaps between air atoms wider. By making the medium more permeable, light facilitates the transmission of effigies. Plato and Aristotle criticised this theory of vision and proposed views that are labelled as *medium* theories since these theories require a medium between eye and perceived thing for vision to be possible. For Plato, human eyes possessed fire similar to sun. These visual rays emanating from eyes mixes with the fire from the sun and creates a "coherent, homogenous and percipient" medium that is necessary for vision to be possible (ibid., 5). In this medium created by the atoms of light emitted by the sun and fire from eyes, the "third fire" from object's surface results in its perception. Aristotle disagreed with Plato about the medium of vision. For Aristotle, the medium is always present and is not created by the "extended sensitivity" of eyes. In this framework, light for Aristotle was the actualised transparency of the medium and this was enabled by the visual fire from the sun and other burning bodies (ibid., 6). In all these theories, "light" is something that makes visual sensation possible. Where they differ from one another is the ontological status of light: for Plato and atomists, light is a real thing — the fire from the sun; however, for Aristotle, light is a particular state of the medium which is actualised at certain conditions. The other set of Greek theories concerning vision — usually categorised as extramission theories — did not indulge in physical explanation of vision, the nature of eyes or light. These theories were concerned about the characteristics of what we see. By starting with the central tenet that visual fire is emanated from eyes, these theories geometrised vision by assuming that the "visual rays" travel in straight line. With this approach, Euclid could describe the apparent dimensions of what was seen based on the angular distribution of visual rays (ibid., 10). This approach was further developed by others including Ptolemy (in the second century CE), who clarified optical phenomena like reflection and refraction.

Medieval Views of Light

Given light was not the primary cause of vision, Greek theories did not extensively explore the physical nature of it. The Arab natural philosophers, who later received the Greek theories, were primarily responsible for bringing light to the centre stage of optics. Ibn al-Haytham (also known as Alhazen), the eleventh century natural philosopher, synthesised "the mathematical sophistication of the physically implausible extramission theories . . . [and] physically plausible, but mathematically naive, intromission theories" to make the optical revolution happen (Bala 2006, 88). Alhazen proposed that the air medium is transparent and light which transmits through this directly causes vision (Darrigol 2012, 17). With this view, he interpreted vision as "light rays" entering the eyes and also identified the role of eyes' lens in image formation. It is only through this paradigmatic shift, light became the central object of study in optics. Alhazen's study of ray optics and experimental methodology was an important factor for the scientific revolution that happened few centuries later. However, this contribution of Alhazen has often been unacknowledged (Bala 2006, 86). For instance, some of the important contributions for optics by Johannes Kepler in the sixteenth century was based on Alhazen's works. Kepler further developed the analysis of eyes' lens and with the analogy of "optical instrument" he argued for the inverted image formed by eyes and other characteristics of the binocular vision (Darrigol 2012, 26-29). Throughout his works, Kepler held that light as " 'emanation' from the luminous points of the source" and described this transmission in terms of "spherical amplifications" (ibid., 31).

2.1.2 Mechanical Interpretations of Light

Post fifteenth century, the study of light was carried out within the then prominent worldview of the natural world — the mechanical worldview. The interpretations of optical phenomena provided by the initial few natural philosophers of this era set the foundation for the discourse on light for the coming centuries, only later to be revised by the electromagnetic worldview. The mechanical worldview, much against the scholastic interpretations preceding it, held that corpuscles or atoms are the fundamental constituents of the natural world and all phenomena could be explained through size, shape and motion of these bodies (Garber 2006, 24). With this predisposition, light was understood as perturbation in a mechanical medium. The prominent natural philosophers during the initial phase of mechanical era (between the fifteenth and the late sixteenth centuries) — Thomas Hobbes, Rene Descartes and Robert Hooke — considered light as "periodic succession of pulses" that are generated by vibrating bodies (Darrigol 2012, 56). It is in the backdrop of this medium theory of light, which was directly influenced by Aristotle's philosophy, that two different interpretations of light emerged — corpuscular and undulatory theories.

Isaac Newton was the leading proponent of the corpuscular theory of light. Unlike his immediate predecessors who were influenced by Greek medium theories, Newton took inspiration from Greek atomists. He was not the forerunner and few natural philosophers before him had already adopted the neo-atomist trend. Under this framework, Newton proposed that light is a "stream of globules" which move quickly in straight lines (ibid., 80). Newton considered that the ability to straightforwardly account for the rectilinear propagation of light through the motion of corpuscles as one of the strengths of his theory. Competing theories that held light as disturbances in the medium could explain this only by resorting to various ad hoc suppositions (ibid., 88). For Newton, chromatic dispersion observed when light passes through prism was the *experimentum crucis* for the corpuscular theory of light. He argued that this observation could only be explained by presuming that light corpuscles possessed the innate quality of colour (Darrigol 2012, 83). This particular claim brings to forefront both physical and metaphysical views of Newton about light. Being very much situated in the mechanical worldview, Newton considered only substances — the basic fundamental units of which are corpuscles — can possess sensible qualities. And since light possessed the quality of colour, it has to be composed of bodies that can bear this quality (ibid., 85). The atoms that compose light are different from the ones that constitute matter and these light corpuscles can only be influenced by forces that are specific to them (McMullin 1978, 83-86). However, in most of his works, the corpuscular interpretation does not get stated and this is particularly due to Newton's style of doing science: he professed that scientific observations and claims should be free from hypotheses (Shapiro 2004, 227). Even though Newton often limited his responses to phenomenological observations, some of his views could only be understood with the presumption that light is a stream of corpuscles (Sabra 1981, 284). In spite of his success in explaining several phenomena, Newton till the end struggled to account for the diffraction phenomenon. The corpuscular description that he proposed for this phenomenon was proven wrong in an experiment he attempted and with these failures, he left the right corpuscular explanation of diffraction to "posterity" (Darrigol 2012, 102).

The diffraction phenomenon was first reported by Francesco Grimaldi in the initial half of the seventeenth century. He observed that light after passing through a small hole casts shadow of an object such that the shadow has several fringes. Since this unique phenomenon could be accounted by using the analogy of "undulations on the surface of a stream", he inferred that light must be like a "fluid" (ibid., 58). Following him, other natural philosophers — like Christiaan Huygens — similarly interpreted light, but the analogy used was that of sound waves travelling in air. According to this view, light is composed of series of compressions and dilations in the medium. Thus, the analogies of water waves and sound waves guided the initial proposals of undulatory interpretations of light. Right from the inception of the mechanical worldview, sound was considered as the quintessential mechanical phenomenon. As sound waves could not be perceived, they were visualised based on the waves on water surfaces. In spite of their centrality, these processes were not rigourously understood during this time (ibid., 37). A stronger version of undulatory theory had to wait until the early nineteenth century. Thomas Young not only provided the right analysis of diffraction and interference phenomena, but also provided the mathematical interpretation of these (Kipnis 1991, 26). Young, from beginning, was against the corpuscular interpretation provided by Newton. During his initial responses to Newton's criticism of wave theory of light, Young utilised the analogy of sound to explain several optical phenomena like rectilinear propagation of light (Darrigol 2012, 169). However, post 1800, Young shifted from the analogy of pulses to that of undulations to interpret light and this enabled him to provide a coherent understanding of interference phenomenon (ibid., 174). With these accomplishments, Young proposed that light should not be understood as longitudinal waves. Instead, light is transverse waves travelling in the medium. It are the positive and negative displacements present in the transverse waves that accounted for the observed interference fringes.[1] After Young, further clarifications provided by Augustin Fresnel about interference phenomenon were crucial for the firm establishment of undulatory theory (Kipnis 1991, 165). Apart from these successes, the observation of polarisation phenomenon and its later explanation made undulatory theory of light more prominent than the corpuscular theory.

The corpuscular and undulatory theories, both of which originated as minor strands in the mechanical worldview, had become dominant competing interpretations of light by the eighteenth century. The debate between these two camps is not just whether light is a wave or a particle, as often depicted. The central contentious point is in fact about the ontological status of light, similar to the ones observed in the ancient Greek and Indian thoughts: is light a real substance or not? The atomists conceived light to have an independent existence and the only way this could be granted within the mechanical worldview was by considering it as stream of corpuscles. On the other hand, the diffusion and undulatory theorists were committed to the reality of the medium and light was reduced merely to a disturbance in the medium.

2.1.3 Electromagnetic Interpretation of Light

The conflict between the two camps continued post Fresnel since the corpuscular view still had few adherents (Achinstein 1991, 24). During the same period, several enquiries were being carried out with regard to the "ether problem". Subsequent physicists and historians have often characterised this problem as pertaining to the role of ether as "an absolute reference system". However, as historian Tetu Hirosige argues, this understanding does not aptly capture the situation. He mentions, "what absorbed the physicists' interest was not the issue of an absolute reference system, but rather the implications of the ether problem for the controversy over the nature of light" (Hirosige 1976, 7). In 1810, François Arago carried out an experiment to see whether the velocities of ether and light differ from one another as this investigation would offer "certain data concerning the true nature of light". According to undulatory theorists, since the movement of waves is determined solely by the medium's properties, their velocity cannot depend on the motion of the source of radiation and would be constant with reference to the medium. In contrast, emission theory dictates that the initial velocity of corpuscles is depended on the source's motion. Given this, Arago carried out his well-known experiment of observing the aberration of light from the stars and concluded that wave theory is more consistent with the observations. With this and few other illustrations, Hirosige concludes that "in the first half of the [nineteenth]

1. The conceptual and historical analysis of Young's interpretation of interference is carried out in detail in section 6.2.

century all those who dealt with the ether problem approached it with the intention of establishing a legitimate theory of light... no one attempted to associate the ether with any privileged reference system for motion" (Hirosige 1976, 12). During this period, the works of James Clark Maxwell further re-emphasised the undulatory interpretation of light. Maxwell, being interested in electric and magnetic fields, was theorising how these forces could be understood through the dynamics of the ether. With the mechanical conception of ether, he derived his famous laws about the fields and recognised that light has to be undulations in the same ether which was responsible for electric and magnetic forces. With this he concluded that light is nothing but electromagnetic waves travelling in ether with a specific velocity (Darrigol 2012, 241). Even though Maxwell's theory brought a significant shift, the paradigmatic change — from light being a mechanical to an electromagnetic phenomenon — was not yet complete. This is because, during Maxwell's time, ether was still a mechanical medium. It took another couple of decades for the complete replacement of the mechanical worldview with the electromagnetic one. Within the mechanical framework, phenomena involving ether were explained using its inertia and elasticity properties. This mode of explanation also covered electromagnetic phenomena like lines of forces, which were understood in terms of vortices and strains in the ether (McCormmach 1970a, 460). The distinction between mechanical and electromagnetic phenomena was recognised and several physicists — like Hendrick Lorentz and Joseph Larmor — attempted to harmonise these two different strands (Hirosige 1976, 25; Darrigol 1994). By late nineteenth and early twentieth century, electromagnetic worldview had completely replaced the mechanical one. In this worldview, ether was the fundamental physical reality and all physical phenomena were reducible to its characteristics. Even the foundational mechanical concepts like corpuscles and mass were reduced to excitations of ether (McCormmach 1970a, 459; Jammer 1997, 150).

2.1.4 Quantum Mechanical Interpretations of Light

In spite of the paradigmatic shift from the mechanical to electromagnetic worldview, the ontological status of light — as undulations in a medium — remained intact. However, both the aspects of this ontological understanding — the wave-aspect and the medium of propagation — were challenged in the next major tectonic shift that happened in the beginning of the twentieth century. There were both old and novel factors that provided the momentum for relooking into these beliefs. The electromagnetic theories about ether and light, which were developed in the previous century, were brought to their conceptual conclusion by Einstein in 1905. With his novel insights about relativity principles, he showed how the electromagnetic characteristics about light can still be understood without the need of the ether. By arguing for redundancy of ether, Einstein showed that light does not depend on a medium (Einstein [1905b] 1989). Einstein in another paper during the same year proposed that light ray is constituted by independent "energy quanta" (Einstein [1905a] 1965). This corpuscular hypothesis of light, which might have historical precedences, was motivated by a twentieth century development: the novel frequency distribution law of blackbody proposed by Max Planck ([1900a] 1967). Planck's law triggered a series of theoretical changes that finally eventualised into quantum mechanics. Among the numerous questions and revisions that this event brought, the one that is of main concern for the present thesis is the wave-particle duality. Since duality will be dealt in the chapter 4, most of the historical and philosophical discussions about radiation during the initial few decades of twentieth century will be considered in detail at relevant points in that chapter.

Before moving on, I wish to highlight an important feature about the way Einstein argued for the reality of light in his relativity paper. To contextualise his argument, it is necessary to note how the reality of light was argued prior to him. During the brief discussion on the classical interpretations of light in (2.1.2), I showed that the debate between the corpuscular and the undulatory theorists should not be merely understood as the difference about the physical manifestation of light — whether light is "wave-like" or "particle-like". Instead, the distinction between their views is about the status of light's reality: unlike the undulatory theory that considered light as disturbances in the medium, the corpuscular theory believed light to be a real substance constituted of its own atoms.

In contrast to these two, a different way of conceptualising light was found in the case of Einstein's theory, which propounds the reality of light without being committed about light's physicality. Einstein, in the well known 1905 paper "On the Electrodynamics of Moving Bodies," argued that the following two stipulations should be considered as "postulates": the principle of relativity and the constancy of speed of light in all frames of reference. With this, he showed that electrodynamics of moving bodies can be explained by presuming that "light ether" as "superfluous" (Einstein [1905b] 1989, 141). Even though the overthrowing of ether has been studied extensively, the immediate implication of this on the status of light and its structure has not been emphasised enough. Prior to Einstein, ether — irrespective of it being interpreted within mechanical or electromagnetic worldviews — was the medium that grounded the wave interpretation of light. With the medium's existence being denied, what happens to the status of the wave-particle debate of classical era? What is being asked here is not which interpretation Einstein favours, the answer to which is clear since in an another paper published prior to this paper, Einstein ([1905a] 1965) does argue for the corpuscular interpretation of light. Instead, does the rejection of the ether's existence imply that the wave-interpretation of light is no more feasible? If this question was posed within mechanical or electromagnetic theories, a straightforward response could have been provided: as both of them were medium theories, denial of the ether would imply that light cannot be understood as a series of waves any more. However, in the context of Einstein's paper on relativity, there is no clear answer since it presumes light to be real with no further qualifications. In fact, the implication of Einstein's proposal on the wave interpretation of light goes unexamined until 1909. In this year, Einstein in a paper clarifies that even though "the theory of relativity has thus changed our views on the nature of light insofar as it does not conceive of light as a sequence of states of a hypothetical medium, but rather as something having an *independent existence* just like matter", "regarding our conception of the *structure of light*, in particular of the distribution of energy in the irradiated space, the theory of relativity did not change anything" (Einstein [1909] 1989, 386). That is, the denial of ether does not necessarily say that light cannot be wave-like. The 1905 "postulates" merely state the reality of light without further commenting on its structure. In this sense, the nature of Einstein's 1905 claim about light's reality is remarkably different from his predecessors. To claim the existence of an entity without being committed about its physical structure was unimaginable both in the mechanical and electromagnetic worldviews. In this sense, as already discussed in (2.1.4), Einstein belonged to a completely new worldview.

2.1.5 Field Interpretation of Light

The development of early quantum mechanics was started off by the non-classical frequency distribution of radiation. However, the theoretical shift brought by this set of events did not remained confined to radiation alone. With the basic fundamentals of radiation questioned, the conception of matter also underwent significant change. Given that the physical interaction of matter and radiation is the central preoccupation of physics, the complete conceptual revision of both the primary physical constituents could have happened in various ways. However, when historically looked, it is interesting to note that it was wave-particle duality question that triggered off this chain of events. By the beginning of the third decade of the twentieth century, Einstein and de Broglie had extended wave-particle duality to matter as well. This universalisation played a key role in the establishment of the quantum field theoretic (QFT) interpretation of both radiation and matter.[2] Works by Born and Jordan ([1925] 1968), Born et al. ([1926] 1968) and Dirac (1927) not only helped in the transition from the "older quantum theory" to "quantum mechanics", but these also established the foundations of QFT. The subsequent physicists worked on these initial formulations to develop the field theories of the present.[3] These developments were also necessary since quantum mechanics had certain deficiencies. For one, it was not clear how to make this theory respect the special theory of relativity. QFT overcomes this and other difficulties. QFT does share the basic aspects of classical field theories. For instance,

2. For a detailed history on the influence of de Broglie's duality proposal on Dirac and Jordan, see Darrigol (1986). In this regard, among several historical narrations of quantum mechanics, the essay by Martin Klein (1964), in which he consciously deviates from the usual route and consider Einstein's duality proposal as the starting point, captures the central role that duality played in the twentieth century physics.

3. For the detailed history on QFT and QED, see Schweber (1994).

in both these theories, "field" of a specific physical aspect (like momentum or energy) consists of assigning values to every point of a space (real or abstract) and these values can change over time in a specific way. However, in QFT, for a field about a particular physical variable, there is another field of the conjugate variable such that these two variables respect the non-commutative principle. This non-classical principle implies quantisation, where the variable can have only discrete values (Kuhlmann 2015).

The field theoretic framework is adopted at present to analyse all the four types of physical interactions: electromagnetic, strong, weak and gravitational interactions. Since the present thesis is about light, a brief extrapolation of quantum electrodynamics (QED), the theory which attempts to analyse electromagnetic radiation and its interaction with matter, would set the ground for further discussions. For illustrating this, consider a perfectly reflecting cavity containing electromagnetic radiation. Within this cavity, certain number of wavevectors are defined such that they form a lattice structure. An electric field can now be defined at all these points within the cavity. The excitation of the entire field can be expressed as the composition of the characteristic of the field at each of the lattice point. For simplicity, the state of the field is expressed in terms of standing-waves, where different states of the field are nothing but different spatial variations ("spatial modes") of the standing wave (Loudon 2008, 12). The energy characteristics of field is described by modelling the standing wave as a quantum harmonic oscillator. The only presumption here is that if the field has to respect Planck's radiation law, the oscillator's energy can vary only in discrete multiples of $h\omega$ (ibid., 12). The energy of the oscillator is given by $E_n = (n + \frac{1}{2})\hbar\omega$, where n takes only discrete values $0, 1, 2, \ldots$ and ω is the angular frequency of the mode. E_n, then, represents the energy of the nth excited state of the field mode. The state of the mode when its energy is E_0 is called the *vacuum state* and its energy will be $1/2\hbar\omega$. Since the energy associated with a mode is quantised, it is possible to express the mode's energy in terms of *quanta* of energy where each quantum of energy is equal to $\hbar\omega$. These quanta are nothing but the *photons* associated with a particular field mode. When the mode of the field increases or decreases, photons are said to be "created" or "destroyed" respectively (Loudon 2001, 9). Starting with this basic picture of radiation, QED attempts to explain most of radiation phenomena by utilising the strengths of both field conception and the mathematical apparatus of quantum mechanics. This theoretical framework has succeeded in not only accounting for the classical phenomena but also has been able to show why semi-classical approaches, where field quantisation is denied, are not sufficient. For instance, QED predicts and explains unique observational phenomena like vacuum fluctuations and quantum beats (Scully and Zubairy 2001, 20-24).

2.1.6 The Age of Photons

As the brief overview of field theory indicates, the notion of "photon" plays an important role in the current understanding of radiation. The prominence of photon is not only due to the dominance of QFT and QED, but has also been actuated by the rise of quantum optics. In spite of the centrality, there is non-clarity surrounding the concept of photon. This confusion is partly contributed by QFT itself as this theoretical framework is ontologically agnostic about which of the two constituents — fields and photons — are fundamentally required for describing the physical reality. Adding to this ongoing difficulty, the rapid theoretical development has resulted in "photon" acquiring multiple meanings. Keeping the discussion about the ontological status of photons for subsequent sections, in the present section, a brief overview of the concept's history is provided.

Photons, according to QED, characterise the energy state ("mode") of the field. This interpretation of photons, first provided by Paul Dirac and Pascual Jordan in late 1920s, was a drastic turn from the conception of light-quantum provided by Einstein. In 1905, Einstein conceived these quanta as independent, spatially localised singularities that constitute the energy of radiation. In 1909, Einstein revised this picture and suggested that radiation consists of waves apart from these singularities ([1909] 1989).[4] Responding to this development, Jordan provided an alternative interpretation in which there is no need to presume these independent discrete energy singularities. Instead, it is sufficient to consider radiation to be waves, whose energy do take on certain discrete values (Born et al. [1926] 1968; Duncan and Janssen 2008). Further, Dirac showed how these photons are "created" and "destroyed" through quantization operators (Dirac 1927; Scully and Sargent 1972). Thus, the concept of photon — what is it and how is it physically? — underwent several revisions within the first few decades of the twentieth century itself. In the early stages of its conception, photon was imagined to be a *point particle* (Kidd et al. 1989, 30). But this model of photon, which can account for momentum transfer, was not sufficient to explain refraction and interference. To account for these, photon was conceptualized as *wave-packets* and *wave-trains*. However, both these conceptions had their problem: they could not account for the compactness of photon and moreover if photons were wave-packets, they would disintegrate over a short duration of time (ibid., 32). These along with few other reasons led to the decline of visualisable physical models of photons. Albeit, Einstein struggled with the physical structure throughout his later career. Even though his working model of radiation was light-quanta surrounded by ghost field, he attempted to conceptualize light-quanta as mathematical *singularities* in the electromagnetic field but felt that the conditions required to achieve these singularities were arbitrary (Dongen 2007; Kidd et al. 1989, 31).

In the middle decades of the twentieth century, the physical interpretation of photons as

4. This conceptual revision is discussed in detail in section 4.2.2.

a classical corpuscular particle faced further challenges. The experimental results conveyed by Robert Hanbury Brown and Richard Twiss in late 1950s created a controversy about the indivisibility of photons (Silva and Freire 2013). The theoretical interpretations provided to resolve the confusion led to the birth of quantum interpretation of photons.[5] Another motivation that also fuelled the transition from classical to quantum interpretation of photons was the resistance shown by the semi-classical theories of radiation towards the idea of quantisation. The notion of photon survived this criticism but in this process the concept itself underwent metamorphosis — it had to shed its classical characteristics to become a full-fledged quantum entity. This transition of photon can be described effectively through the discussion of Clauser (1974), one of the earliest work on the quantum characteristics of photon. The central intention of Clauser is captured succinctly by the title of the paper — "Experimental distinction between the quantum and classical field-theoretic predictions . . . ". The principle way QFT deviates from the classical field theory (CFT) is with respect to the quantisation of the field's energy. As Clauser mentions, there were no conclusive experimental verification that could clarify whether radiation field is actually quantified or not. In order to achieve this, Clauser conducted an experiment where the correlation behaviour of light radiated from the photoelectric effect was analysed. The experimental setup was such that the observation could positively say whether CFT or QFT were correct. This is because CFT and QFT predicted different correlation behaviours of photo-detectors. According to CFT, the correlations observed in this kind of experiment should respect Cauchy-Schwarz inequality. Validation of the Cauchy-Schwarz inequality observationally implies that detectors would show coincidence detection. Contrast to this, QFT predicted that this inequality would be violated (ibid., 857). Therefore, if QFT is correct, then the detectors would not be activated at the same time. The non-coincident activation of detectors, according to Clauser, physically signify the indivisibility of photon. Regarding this particle-like behaviour of photons, Clauser mentions "A particle must be either transmitted or reflected. Both may be done simultaneously only by a wave. We then see how these macroscopic features of 'particle-like' objects arise from the QFT formalism" (ibid., 854). The central aim of the experiment reported by Clauser, therefore, is to exhibit this characteristic feature of photon. Regarding this, Clauser mentions "a photon is not split in two by a beam splitter is certainly 'old-hat' and it may seem surprising that we have gone to the effort to test this prediction experimentally. What is in fact much more surprising is that evidently no such experimental test has heretofore been performed, and such tests are clearly of great importance" (ibid., 855).

Experiments favouring QFT were outcome of an intense period of theoretical debate between the two camps.[6] The pioneering works of Roy Glauber (1963a, 1963c, 1963b) set

5. This controversy and the theoretical discussions play an important role in the clarification of wave-particle duality of single photons. A detailed discussion of these events will be carried out in section 5.5.

6. For more on this, see Bromberg (2016).

the foundation for the right way to think about radiation using QED interpretation of photons.[7] This and various other experimental achievements led to the eventual birth of quantum optics (henceforth referred as QO). In this phase of developments, various non-classical behaviours of light were observed (Loudon 1980). Thus, only with the advent of QO, clearer methods of generating radiation in single photon state (from now onwards will be referred as $SPSR$) were recognised. During this period, as mentioned while discussing Clauser's experiment, there were ways to evaluate whether the generated light is in single photon state. When SPSR is made to pass through a beam splitter, unlike the classical light, it does not get split and go in both the directions. So, if two detectors are placed on either side of the beam splitter, only one of the detector registers a photon. This is usually referred to as the anti-correlation effect of SPSR. The developments of maser and laser propelled the technical possibilities of generating radiation of various intensities. With these capabilities, at present, it is possible to generate "single photons on demand" (Grangier and Abram 2003; Lounis and Orrit 2005). The quantum optics revolution clarified that light-beam conceived just as a stream of photons is too simplistic a picture since the actual characteristics of light depends on the way photons are distributed across the beam. Accordingly, there are two methods of characterising photons' distribution in a beam: either through photon statistics or using correlation functions (Fox 2006, 117). Both these methods provide ways of differentiating non-coherent light from coherent light. Classical optics too distinguished between these two kinds of light; but the distinction was drawn on the basis of intensity fluctuation over time and did not depend on the premise of photons. Apart from these two kinds of light, QO describes a new kind of light-beam which cannot be meaningfully articulated within the classical realm (ibid., 82). This new kind of light is characterised by the even distribution of photons across the beam such that it is possible to deal with just a single photon at a given moment. Regarding the classification of these three kinds of light, the statistical approach categorises them as *super-poissonian*, *poissonian* and *sub-poissonian* light respectively (ibid., 82). When correlation functions are used, these kinds of light are labelled as *bunched*, *coherent* and *antibunched* light (ibid., 117).[8]

The brief tour about the evolution of photon concept highlights how within a period of a century the concept has gone through various restructuring. Coined by G. N. Lewis (1926) in a completely different context, "photon" has come to mean a wide variety of things. For instance, in a recent book, Klaus Hentschel discusses twelve "semantic layers" of photon to indicate how this concept has accrued various meanings through the passage of time (2018, 39).[9] He identifies the following sources through which "photon" (or equivalently

7. These new theoretical insights will be considered in detail during the analysis of interference phenomena of photons in the chapter 6.4.

8. These two classifications are not equivalent to each other. Being antibunched does not necessarily imply that light is sub-poissonian as well (Fox 2006, 117).

9. Here, Hentschel uses "layers" to indicate contexts which uniquely define a concept in a specific

"light-quantum") has derived its multiple connotations:

1. The corpuscular theory proposed by Newton and its later reprisal by Einstein
2. Light having the property of finite velocity of propagation
3. Radiation emission and absorption by matter in the form of light quanta
4. Light ray (similar to other rays) exerting pressure on target
5. Energy transfer by light to matter
6. Discovery of light's energy being proportional to its frequency (through photoelectric effect)
7. Radiation's energy being quantised
8. Light having both wave and particle features
9. Phenomena of spontaneous and induced emissions
10. Discovery of Photon's spin
11. Indistinguishability of light quanta
12. Photons being the virtual exchange particles in QED

As can be seen, each of these "layers" attribute a specific characteristic to photon. Given this tumultuous history, it is not a surprise that there is considerable confusion about this concept. Several physicists have taken notice of this and have reacted sharply. During the rise of QO, there was a suggestion to impose a "ten year moratorium" on the usage of "photon" (Hentschel 2018, 149). W. E. Lamb (1995), in an aptly titled article "Anti-Photon," laments on the misuse of "photons" in modern textbooks. The situation in the recent decades has not improved. Roy Loudon including an introductory note on photons in the latest edition of his book *The Quantum Theory of Light* clear indicates the despair and confusion. He starts this introduction by noting that "The use of the word 'photon' to describe the quantum of electromagnetic radiation can lead to confusion and misunderstanding. . . Nevertheless, the word is extremely convenient and its avoidance often leads to lengthy circumlocutions. The adoption of the photon by the quantum-optics community is widespread and the present book follows current usage, with sometimes imprecise statements that could amount to misuse of the word. The intention of this Introduction is to limit the damage that might otherwise occur by briefly explaining the concept of the photon as used in the text." (2001, 1). In the present context of ambiguity, the following comment from Arthur Compton in 1928 regarding the preference of usage of "photon" is relevant: "In referring to this unit of radiation I shall use the name 'photon' suggested recently by G.N. Lewis. This word avoids any implication regarding the nature of the unit, as contained for example in the name 'needle ray'. As compared with the term 'radiation quantum' or 'light quantum', this name has the advantages of brevity and of avoiding any implied dependence upon the much more general quantum mechanics, or

way. With the usage of geological analogy to describe how concepts accrue multiple semantic meanings, Hentschel thinks layering metaphor helps in describing various characteristics of "complex, nonlinear processes of accretion" (2018, 6).

upon the quantum theory of atomic structure" (Hentschel 2018, 34). Thus, it appears that Compton's intention to use neutral term seems to have over served its purpose and has given rise to an situation where it is no more clear what "photon" means.

2.2 Interpretations of Photons

The above discussion illustrate how the nature of light — not only the reality status and the physical structure, but the explanation of phenomena — is uniquely characterised by a theory. Since light cannot be perceived directly, and can only become aware of its presence through things it illuminates, interpreting optical phenomena let alone light itself requires the mediation of theory. Given this dependency of conceiving light on theory and method of exploration, in this section, I want to discuss two prominent conceptions of photons: the theoretical and operational notions of photons. With regard to the theoretical notion, I will discuss Dirac's interpretation[10] in sub-section 2.2.1 and an important question this interpretation raises — the ontological status of photons in field theories — in sub-section 2.2.2. The operational notion is discussed in sub-section 2.2.3.

2.2.1 Notions of Light-quantum in Dirac's Analysis

In his 1927 paper, Dirac argued that both wave and particle interpretations of radiation are equivalent in nature. In this work, two different notions of light-quantum are encountered: (i) the independent "light-quanta" which constitutes light and (ii) the discrete "quantum of energy" that radiation possesses. For Dirac, both these are independent notions. As he mentions, if the hamiltonian of radiation is analysed with energy and phase as non-commutative variables, then this necessarily provides the much required quantisation of radiation's energy, the "energy-quanta". Apart from the notion of discrete quantity of energy, Dirac uses "light-quanta" notion to discuss Einstein's analysis of emission and absorption of radiation. This second notion of light-quantum initially seems to be similar to that of Einstein since Dirac also considers them as mutually independent entities having momentum, a state of polarisation and are characterised by Einstein-Bose statistics (Dirac 1927, 260). However, further characterisation of quanta that is observed in the paper are very unique to Dirac's interpretation. For Dirac, light-quantum can be in a state — called *zero state* — where it does not possess any energy or momentum (ibid., 260). The possibility of this theoretical state allows for him to talk about "creation" and "annihilation" of quanta: "when a light-quantum is absorbed it can be considered to jump into this zero state, and when one is emitted it can be considered to jump from the zero state to one in which it is physically in evidence, so that it appears to have been created" (ibid., 261). Light-quantum thus conceived does not seem to be physically equivalent to

10. Einstein's notion of light-quanta will be discussed in section 4.2.

Einstein's light-quantum. Even though, light-quantum for Dirac still plays the crucial role in the phenomena of emission and absorption of radiation, the characterisation of quanta in zero-state has certain peculiarities, like for instance, "since there is no limit to the number of light-quanta that may be created in this way, we must suppose that there an infinite number of light-quanta in the zero state" (Dirac 1927, 261). Helge Kragh comments that Dirac introduced these "unobservable, or spurious photons ... with zero energy and momentum ... because he believed such entities were necessitated by formal reasons" (Kragh 1990, 123).

Both notions of quanta are represented in Dirac's paper as "N_r". In the wave interpretation of radiation, N_r stands for the number of energy-quanta associated with a particular component of radiation r (Dirac 1927, 262). Contrast to this, in the light-quanta interpretation, N_r represents the number of light-quanta in state r (ibid., 261). Therefore, both these meanings of N_r are never invoked simultaneously. Given this, with respect to the equivalence between both the wave and the particle interpretation of radiation, "the number of *particles* ... which can be used as a dynamical variable in the Hamiltonian for the particles is equal to the number of *quanta of energy* in the corresponding wave in the Hamiltonian for the waves" (ibid., 245, emphasis added). Even though N_r gets interpreted either as (i) or as (ii) depending on the context of interpretation, the notion of energy-quanta is more fundamental than the notion of light-quanta. Given this, in the particle interpretation of radiation, we see both the notions of quanta coming together.

The presence of two notions of quantum found within the same theory of Dirac seems like the final assimilation of the two parallel views — of Planck and Einstein's — on "elements of energy", which will be discussed in (4.2). To rephrase this using Ehrenfest's terminology, Dirac's analysis finally brings together the "quantisation" and "corpuscularity" aspects of light-quanta. Here, it should be noted that the assimilation of these two aspects is not at the entity level, as Einstein proposed. In the 1905 paper, Einstein argued for the physical entity of light having the discrete quantity of energy. In contrast to this, Dirac's paper brings together these two concepts within the framework of a single theory. However, he could achieve this only by holding onto the equivalence between the two ways of analysing the radiation-matter interaction.

2.2.2 Ontological Status of Photons

Another important strand of development regarding the theoretical interpretation of photon is contextualised in the debate about QFT's ontology. The quantum field theory is peculiarly ambivalent about its basic ontology since the theory can be interpreted equally from the particle and the field viewpoints. However, this does not imply that both the interpretations enjoy equal status. The notion of photon faces formidable challenges within QFT framework. Even though field theory does require the notion of quantisation, the problem arises when these quanta of energy are interpreted as independent entities. One of the difficulties of considering photons as genuine particles is that the definition of the state of a photon in QED at a particular position in space is not possible. To understand this, consider the case for fermions like electrons and mesons. Through the process of second quantisation prescribed in QED, fermions can also be understood through quantised fields. In this framework, the state $|\psi\rangle$ of a fermion in a specific position r, which can be represented as the eigenstate $|r\rangle$, is understood as the creation operator of the fermion field for the specific position — $\hat{\psi}(r)$ — acting on the vacuum state of the field. This is usually represented as $|r\rangle = \hat{\psi}(r)|0\rangle$. A similar procedure, however, cannot be executed for bosons like photons (Scully and Zubairy 2001, 28). This inability by itself has been considered as a good reason for not considering photons as real entities. Adding to this difficulty, with no way to assign position, photons can only be characterised by the three fundamental properties: energy, momentum and polarisation (Zajonc 2008, 8). And this results in, as mentioned during the discussion of quantum particles in general, the non-distinguishability of photons since all of them in an ensemble can have the same values for all the three properties. With there being no way to assign identity over time, photons have been considered as "mere appearance" and not genuine entities (Kidd et al. 1989, 32).

Several arguments regarding the problems of considering photons as real entities have been offered (Malament 1996; Halvorson and Clifton 2002; Fraser 2008). Regarding the abstract nature of photons, a recent work argues that "the way quanta are introduced within the mathematical formalism of QFT does not support the claim that they are ontologically robust entities resembling particles in any significant sense" (Bigaj 2018, 148). The author, Thomasz Bigaj, in order to substantiate this point, focuses on the analysis of simple harmonic oscillator (SHO) since in QFT, the energy of field and particles are modelled on it. Bigaj mentions how "quanta of energy" are as usually interpreted as "entities" that are "produced" by this oscillator (ibid., 148). By providing several arguments against this concept, Bigaj argues that these are not genuine physical entities. To begin with, he mentions that energy quanta of SHO are not "subjects of property attributions . . . if anything, they are energies . . . the energy is still attributed to the underlying physical oscillator" (ibid., 149). Even though it is claimed that quanta are countable, these entities are incapable of possessing "possible properties". Moreover, Bigaj claims that "creation" and "annihilation" operators are just "metaphorical way of speaking . . . with no deeper metaphysical meaning" (ibid., 149). Bigaj further analyses Fock space formalism and other aspects of QFT that yield support to particle interpretation and in each of these cases, shows the problem of particle interpretation. Thus, Bigaj thinks that photon is "mere bookkeeping device" and should not be considered as ontologically real (ibid., 148).

2.2.3 Operational Interpretations of Photons

The above discussion about anti-realism of photons is situated in the theoretical interpretations of QFT in general. In contrast to this tendency to dismiss these entities, the realistic commitment towards them can be seen in quantum optics (QO). As discussed in section 2.1.6, the rise of QO enabled not only new interpretations about the nature of light, but brought novel experimental observations exhibiting the presence of photons. This importance of photons for QO is indeed surprising given that QO's theoretical lineage is traced to QFT. So, with these two large active sub-disciplines of physics having different allegiance towards photons, it is difficult to comment on the reality status or even the pragmatic need of these entities. For instance, Bigaj recognises this conflict by noting that "an additional physical motivation for calling the excitation $\hbar\omega$ a quantum can come from the well-known hypothesis of the corpuscular character of electromagnetic radiation as embodied in the notion of a photon. However, the corpuscular character of electromagnetic waves can be supported by experimentally verified phenomena, such as the Compton scattering and photoelectric effect. To my best knowledge, no similar arguments exist in the case of a single harmonic oscillator" (Bigaj 2018, 149n20). It is evident from Bigaj's comment that the argument that he is drawing based on SHO is limited to the general argument of QFT and would be far fetched to directly apply in the cases of QO. With the distinction in use of photons in QFT and QO highlighted, in the present section, I want to discuss few attempts to provide the operational notions of photons.

The primary aspect that needs to be discussed is how to talk about photons in the experimental context. Regarding this, Loudon (2001) provides an initial clarification. In "open optical systems", Loudon thinks that radiation cannot be understood through the models that were used in the theoretical analyses. As discussed in the section 2.1.5, in QED, radiation inside a cavity is theoretically interpreted in the form of discrete standing waves. In contrast to this, in instrumental setups like Mach-Zehnder interferometer, light has to be analysed as "discrete travelling-wave modes that propagate from sources to detectors" (ibid., 1). Here, Loudon defines "spatial mode" based on the "geometry of the apparatus" and hence would include "input light waves, both paths through the interferometer, and output waves". The spatial mode is defined in this form because the single photon excitation is considered to be "distributed over the entire interferometer". The above interpretation of the spatial modes in open systems, which use single-photon-state light sources, is "not strictly valid", but as Loudon qualifies this way of thinking "has acquired respectability from some distinguished contributions to the discussion ..." (ibid., 2). With respect to light emitted from the source, Loudon suggests to interpret the "photon excitations" in terms of *spatial wavepacket function*, which is "an integral over contributions from waves with a range of frequencies" (ibid., 2). Given that a wavepacket is constituted by waves that range over a continuum of frequencies, the energy associated with such a wavepacket

— the "photon" — would be the mean energy that is the product of $\hbar$ and the average of the frequency components (Loudon 2001, 2). Loudon further mentions that "the level of excitation of the system continues to be represented by a number operator with integer eigenvalues". Given this, when a suitable radiation source is used, it produces "a single current pulse in the ionization of a photodetector" (ibid., 2). Because of this, Loudon affirms that the notion of photon "survives as an operational definition" and "it provides a useful qualitative description of the nature of the state" (ibid., 2).

The above guidelines by Loudon shows how, in spite of the theoretical difficulties for committing to the reality of photons, we can use carefully the language of photons to talk about experiments. With this initial prescription, for every scenario and setup, the proper way of talking has to be figured out. To illustrate this, consider the discussion by Scully and Zubairy (2001, 28) on interpreting the presence of "single photon" at a particular place. Here, the authors suggest that instead of asking the question what is the state of a photon at a specific location, it is better to ask the equivalent *operational question*: "what is the probability that a single-photon state of the radiation field ... will lead to the ejection of a photoelectron by a detector (atom) placed at point r?". Here, the "single-photon state" of the radiation field is given by

$$|\psi\rangle = \sum c_{\{n\}}(t)|\{n\}\rangle \tag{2.1}$$

where $\{n\}$ represents all possible states of the field such that each of these states have modes that are in single-photon state (ibid., 28). Detecting a single photon at the position r using an atom is equivalent to a photon getting annihilated by that atom. By reinterpreting the detection of the photon through the annihilation of it by the atom, the authors define the field equation of photon. In this description, the structure of the field becomes similar to the one used for detecting a neutrino. The similarity between the two equations seems to show a way to provide the particle interpretation for photons. However, when this equation for photon is compared with electron, the particle interpretation of photon breaks down (ibid., 33). In spite of the above difficulties, the authors suggest a scenario where operational notion of photons can still be defined. In a situation where we have two-photons source and two detectors, the probability of the detectors detecting each of these photons depends on the two-photon correlation function (ibid., 33). This correlation function depends on the atomic decay rates of these photons. The usual picture of two stage atomic decay requires three levels — first the atom decays from level $|a\rangle$ to level $|b\rangle$ and from this level the atom decays to level $|c\rangle$. Each of these decay processes can be quantified through the decay rate — γ_a and γ_b. These decay rates are important because if $\gamma_a \gg \gamma_b$, then the correlation function gets the structure of detecting two distinct particles. If this is not the case, then "two-photon detection amplitude" does not have the right form. With this, the authors suggest that when $\gamma_a \gg \gamma_b$, it is possible to speak as if "independent photons" have been emitted. However, when the scenario becomes such that $\gamma_a \ll \gamma_b$, " 'photon-as-a-particle' picture is very misleading" (Scully and Zubairy 2001, 35).

Chapter 3

Different Notions of Duality

Before starting with the analysis of wave-particle duality of light, in this chapter I intend to do a survey of the general notion of duality found in various disciplines like mathematics and sciences. Duality and its cognates (duals, dualism, etc.), generally speaking, refer to a situation where there are two things involved. However, the pair of things involved do not decide the notion of duality completely. The characteristic relation that constitutes this duality plays an important role too. Thus, in this chapter, the principle aim is to explore the different notions of duality and how each of these concepts are unique. To begin with, duality found in logic, projective geometry and vector algebra are extensively studied. By analysing the concept of duality found in each of these scenarios, I will also attempt to address few important questions about how to interpret duality in these cases. Following to that, duality found in other areas like group and category theories,[1] dual theories of physics and in other branches of science are highlighted. Finally, I will attempt to briefly explore the overlap between the concepts duality and symmetry.[2]

3.1 Basic Forms of Duality

Cardinal Duality

To begin with, consider the simple notion of duality which involves just two things conjoined by connectives like “and”, “or” or even a hyphen. This description of duality is not specific about the relation that binds two things and hence works with the minimum requirement that there has to be two things. However, owing to the scarcity of specifications, this type turns out to be an uninteresting form of duality. This is largely because these connections

1. The pervasiveness of duality in mathematics has been observed and discussed by several mathematicians (see Atiyah (2007)). Also, several philosophers of mathematics have attempted to provide unified theories of these multiple duality instances found in different branches of mathematics using category theory (see Corfield (2017)).

2. Parts of this chapter appear in a paper co-authored with Sundar Sarukkai. See Bhatta and Sarukkai (2020).

between things are not restrictive enough as they can stand between any two things, even when these things are not of the same category (i.e., one can be an object and the other can be a quality). In spite of this triviality, this basic notion of duality captures an aspect which is central to the notion of duality and that has to do with *duals* — there being just the right number of things, not one or three. In this basic form, therefore, what matters to duality is nothing more than the count of things involved irrespective of how these two things are involved, which might be through conjunction, disjunction, or any other form. In this form, the notion of duality is on the verge of reducing to a trivial notion as there is nothing significant about the count two and there are innumerable contexts in which there are two things involved. Moreover, any two countable things can be arbitrarily brought together as a dual. Even though the relations involved connect just these two things, the things that are brought together might have nothing specific to do with being dual. This also implies that if the pair "A and B" (where A and B can be anything) are dual of this trivial kind, then there is nothing about A and B that restricts the possibility of joining another thing and forming "A and B and C". Because of these reasons, this simpler version of duality turns out to be too ad hoc and arbitrary.

This non-significance of cardinal duality is largely due to its bare minimum characterization and it is evident that something more is required to make it a more meaningful concept. The subsequent sections will show conceptually interesting variants of duality present. But before discussing them, I will illustrate a variant of this cardinal duality that we commonly use while cataloguing and classifying the world around us. In this slightly restrictive form, rather than referring to a collection of any two specific things, like the dual moons of Mars, this duality can be used in a bit abstract form to refer to the (cardinal) duality of specific kinds of things that we regularly see around. Usage of duality in this context, as can be seen in the cases like "pair of scissors", attempts to capture things that usually come in pairs. A well known example of this kind is the presumption of dual biological sexes: male and female. As Oudshoorn (1990) discusses, the underlying presumption of "sexual duality" shaped the development of biology in the last two centuries. It is only in the twentieth century that this "sexual duality" gets challenged. A not so popular, although historically interesting, instance of this usage pertains to the "duality of brain" and with that the "duality of mind" as well. The anatomical discovery of human brain having two hemispheres (during the nineteenth century) gave raise to the speculation that we do not have a single, unified brain. Instead, we possess "dual brains" and "dual minds" (Puccetti 1989). Briefly, according to this proposal, it is "entirely unphilosophical . . . to speak of the cerebrum as one organ. The term two hemispheres of the brain is indeed, strictly a misnomer . . . The two hemispheres are really and in fact two distinct and entire organs, and each respectively as complete . . . and as fully perfect in all its parts . . . as are the two eyes" (ibid., 138).

Duality as Opposition

The above discussion shows how the basic interpretation of duality becomes uninteresting since it merely reduces to specifying the count of things involved. However, apart from the bringing together of two entities to constitute a pair, there is another conception of duality that is commonly encountered. Even though this conception of duality also involves two entities, these things are contrary to each other. Often mentioned instances of this kind include duals like “good and bad”, “true and false” and “white and black”. This duality can be illustrated through the duality of human sexes mentioned earlier. In the early twentieth century, during the initial stage of sex endocrinology as a new field of research, sex hormones were classified into two categories not because the identified hormones fell into two distinct kinds (similar to the discovery of two hemispheres of brain). Instead, “this dualist conception can be seen as a biological translation of the still-influential Victorian ‘Doctrine of the Two Sexes’ which held that women’s activities were in most respects *opposite* to those of men. Female and male were understood as opposite categories, not as two independent or similar dimensions” (Oudshoorn 1990, 169, emphasis added). Unlike cardinal duality, which was mere conjunction of two entities, the nature of this duality is not directly evident and understanding this relation demands analysis of several interrelated concepts like negation, opposition, complementarity, and contrariety.

Beyond Superficial Dualities

Contrast to the above mentioned general usages, there are other invocations of duality found in logic, mathematics and physics, where this notion is used in much more interesting ways. These specific instances of duality, however, appear schematically in forms that are not different than the ones found in the basic usage. For example, the dualities like “conjunction and disjunction operators” or “wave-particle” look as if the relation between the duals is nothing more than the simple connection that was discussed above. In spite of the superficial similarity, these instances of duality are much more complex. In fact, as I will show below, the entire task of understanding these dualities involve interpreting the seemingly straightforward connectors (‘and’, ‘or’, ‘-’) that exist between the duals. This characteristic of the relations being more than what they appear can be illustrated through a relevant phenomenon found in several languages. To articulate this within English language, consider the common phrases like “salt and pepper”, “more and more”, “good or bad”, “by and large”, “mother and kid”, “life and death”, etc. These phrases are not too unfamiliar as they are encountered regularly in our everyday linguistic exchanges. However, these are not instances of simple duals as the relation between the words in each pair is a complex one. Labelled as *binomials* in linguistics, this kind of phrases consists of two words that are related not only by some syntactical relationship but they also share several semantic and phonological relationships between them (Benor and Levy 2006). These relations shape important characteristics of binomials like the order in which the pairs appear, the flexibility of the order, etc.

As the above discussion illustrates, there is a need to go beyond the obvious surface-level relation that appears between the duals. Thus, when "wave-particle duality" is invoked, it is not obviously clear how 'wave' and 'particle' are related by the connector. The complex nature of duality relations is illustrated in this chapter by discussing various dualities found in mathematics and sciences. As will be shown, these instances of duality are much more than just juxtaposition of two entities. Also, it is important to notice how each of these instances have a unique duality relation.

3.2 Duality in Logic

3.2.1 Duality between Truth-Functions

In logic, a specific relation amidst truth-functions has been labelled as duality. *Truth-function* is a statement whose truth-value (i.e., whether it is true or false) depends on the truth-values of the constituent statements and how they are connected through logical connectives like AND, OR, NEGATION, etc.

Duality of Conjunction and Disjunction

The best way to introduce this kind of duality is by highlighting how simple conjunction and disjunction truth-functions (i.e., statements constituted by the AND and OR operators) instantiate this duality. These statements are represented schematically as $p \wedge q$ and $p \vee q$ respectively, where p and q are statements. These schemata are considered *duals* of each other because the truth-table of these functions, which usually consists of the possible truth-values of the functions for different truth-values of their constituents, show a specific type of similarity.

p	**q**	$\boldsymbol{p \wedge q}$
T	T	T
T	F	F
F	T	F
F	F	F

p	**q**	$\boldsymbol{p \vee q}$
T	T	T
T	F	T
F	T	T
F	F	F

Table 3.1: Duality of $\wedge$ and $\vee$

As can be seen in Table 3.1, the similarity between these two statements' truth-tables is not directly evident. However, these truth-tables are related under specific transformation.

To demonstrate that, consider, the table of conjunction. In this, if the truth-values are swapped in all the columns, we end up with a table that has the truth-value distribution as shown in Table 3.2.

F	F	F
F	T	T
T	F	T
T	T	T

Table 3.2: Truth table of conjunction after swapping T and F

This resultant table, when few rows are rearranged, is nothing but the truth-table of the disjunction operation shown previously. Therefore, even though the disjunction and conjunction schemata have different truth-tables to begin with, these tables turn out to be similar to one another under a specific transformation. This indicates some structural similarity between these two schemata. So, it is the simultaneous consideration that these two truth-functions are unique and yet are similar to one another after some alteration which is being labelled as *logical duality*. According to Quine, duals are "*alike* under truth-value analysis except for a *thoroughgoing interchange*" of T and F (Quine 1956, 59, emphasis added).

Duality and Negation

The above transformation specified for arriving at the dual of a schema appears very similar to carrying out a negation operation which, by definition, switches the truth-values. So, is the transformation specified equivalent to carrying out the negation operation and does negating a schema gives its dual? Simple negation does interchange the truth-values of a schema, but this interchange is confined only to a particular column. The other parts of the truth-table do not come under the scope of this operation. To demonstrate this through an example, see the negation of $(p \wedge q)$ in Table 3.3. Here, the negation operation just alters the value of the final column and the other columns in the truth-table (like that of statements p and q) are unaltered.

p	**q**	$\boldsymbol{p \wedge q}$	$\boldsymbol{\sim (p \wedge q)}$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Table 3.3: Truth table of conjunction after negation

Negation, therefore, does not bring about a "thoroughgoing interchange" of values of a truth-table that is essential for invoking duality (Quine 1956, 60). But this still might not be completely convincing as the above illustration involves a complex schema that has components. So, negation carried out on a particular part will not affect other parts. In contrast to this, it might be suggested that if a simple statement is considered, the negation of that will completely interchange the statement's truth-value. So, at least in the case of simple statements, can p and $\sim p$ be considered as duals of each other? In order to evaluate this suggestion, consider the truth-table of negation as shown in Table 3.4.

p	$\boldsymbol{\sim p}$
T	F
F	T

Table 3.4: Truth table of simple negation

When negation operation is carried on the values of p, which are given in the first column of the table, the outcome is the other column represented under $\sim p$. And this is true the other way round as well: starting with $\sim p$ column values, negation operation will provide column p values. Given this, the negation operation seems to interchange the truth-values. However, since the execution of this operation brings the interchange of values in only one column, the outcome is not a thorough going interchange, which is essential for duality.

Self Duality of Negation Operation

The above exercise brings out an important aspect of logical duality. Since the duality is among two truth-functions, the accompanying transformation is not limited to specific columns, but pertains to the entire truth-table. Moving on, consider the dual truth-function of a simple statement that is negated. The analysis of this scenario brings out an important characteristic of logical duality: the aspect of self-duality. To demonstrate this, let us carry out the specified procedure for duality: start with the truth-table of negation as shown in Table 3.4 and thoroughly interchange its truth-values. With that, the table gets transformed as shown in Table 3.5. The resultant table, surprisingly, is again a truth table of negation operation. This implies that the dual of negation operation is negation itself (ibid., 60).

F	T
T	F

Table 3.5: Truth table of simple negation after thorough interchange

3.2.2 Generalized Procedure

Through these examples, it can be seen that every logical proposition has a dual. This availability of dual for every schema indicates that logical duality is not some unique characteristics of some few schemata. Instead, duality transformation is something that can be performed on any schema to reveal its dual. Given that every truth-function has its dual, is there any general procedure for arriving at the dual of a schema? One such procedure has been mentioned above: interchange thoroughly the values in the given schema's truth-table to arrive at the dual. Apart from this method, there are few other procedures to arrive at the dual of a schema and all of these, contrast to dealing with the truth-table of a schema, work with its symbolic representation.[3] One of the procedures, which works for schemata that have only $\wedge$ and $\vee$ operators, involves replacing all disjunctions present in the schemata with conjunction and vice versa (Quine 1956, 61). This procedure is generalization of the duality observed between $(p \wedge q)$ and $(p \vee q)$ discussed above. To consider a different instance, this procedure carried out on the schema $(p \wedge q \vee r)$ gives $(p \vee q \wedge r)$ and when the truth-tables of these two schemata are considered, the duality between them becomes evident.

An alternative procedure is more generalized than the swapping of $\wedge$ and $\vee$ and works for any statement. According to this, the dual of a schema can be found by negating the schema as a whole along with the negation of all its individual components. This procedure is in fact translation of the procedure of thorough-going exchange carried out on the truth-tables into symbolic operations. Moreover, this procedure further clarifies the discussion on negation and duality. Mere negation of a schema will not provide its dual; instead, negation of the individual components is required along with the negation of the whole. To provide an example of this, execution of this symbolic operation on the truth-function $(p \wedge q \vee r)$ yields its dual as $\sim (\sim p \wedge \sim q \vee \sim r)$.

It does not come as a surprise that there are multiple procedures to arrive at the dual of a schema. One procedure operates at the truth-table level; the other two are symbolic manipulations. In spite of the differences in the execution steps, all these three procedures have the same central characteristic — the procedure can be recursively applied on the resultant to get back the initial schema or truth-table. For instance, given a truth-table T, the procedure of swapping the truth-values results in its dual T'. The same procedure — of swapping the truth-values — when repeated on T', gives back T. This is the same case with the procedures that modify the operators in the symbolic representations of the schemata. Therefore, this feature of the procedures to "complete the circle" — starting from S, giving S' and resulting in S again — captures in an intuitive way the essence of being "dual to each other". The repetitive implementation of these procedures toggle between the dual pairs — S and S' and the respective truth-tables T and T'.

3. Quine, in fact, labels these procedures as "laws of duality".

3.2.3 Duality and Equivalence

There being two generalized procedures of arriving at the dual of a schemata raises an important query: how is that for a given schema, say $(p \wedge q \vee r)$, there can be completely two divergent ways of arriving its dual? Moreover, these two procedures not only differ in the process of generation, each of these give rise to a dual that is different from the other: one gave $(p \vee q \wedge r)$ and the other resulted in $\sim(\sim p \wedge \sim q \vee \sim r)$. So, how is that for a particular schema, there can be two different duals? Now, even though $(p \vee q \wedge r)$ and $\sim(\sim p \wedge \sim q \vee \sim r)$ are different in the structure of their components, when both these schemata are evaluated for similar truth-values of the components, they both give the same result. This shows that these two apparently different schemata are in fact *equivalent*. Two truth-function schemata are said to be equivalent if "they agree with each other in point of truth value under every interpretation of their letters" (Quine 1956, 46). Therefore, these two schemata are not different after all and this equivalence among them resolves the confusion about different representations of the "dual" of a schema. Speaking generally, if there are two equivalent schemata S_1 and S_2, then their duals — S'_1 and S'_2 — will also be equivalent to each other (ibid., 62). This implies that the duality transformation respects the equivalence relation: the duality transformation of S_1 to S'_1 makes sure that S'_1 will also be a dual to S_2. This can be expressed the other way round as well: if S'_1 is the dual of S_1, then an equivalent truth-function of S'_1 — say S'_2 — subjected to a thoroughgoing transformation results in a truth-function S_2 which will be equivalent to S_1. The relation between equivalence and duality can further be analysed by exploring what happens to duality in the context of logical implications, since equivalence is nothing but a particular type of implication, i.e., mutual-implication. Symbolically, if two schemata are related to each other through implication, like for instance $S_1 \rightarrow S_2$, then how would their duals — S'_1 and S'_2 — be related to each other? It can be shown that in these cases, the implication still holds good but with a slight variation: the direction of implication is reversed (ibid., 62). Thus, symbolically, $S'_2 \rightarrow S'_1$.

As the above discussion has highlighted, there seems to be a conceptual overlap between duality and equivalence relations. To understand this further, consider first two truth-tables T_1 and T_2. These two truth-tables are equivalent if they have the same truth-values at every step. Contrast to this, when I compare T_1 with its dual T'_1, then these two truth-tables disagree with each other at every step. It appears, then, that duality and equivalence are related aspects of truth-tables. Both are specific attributes that qualify the comparison of truth-functions. If equivalence brings out the complete similarity among the truth-tables, duality, on the other hand, indicates that the two tables are opposite to each other in every step. Therefore, equivalence and duality seems to be conceptual antonyms of each others. Understanding duality as a non-trivial antonym of equivalence helps in understanding why Quine thinks that dual tables are "*parallel* in their behaviour" in spite of "structural similarity" between them (Quine 1956, 59, emphasis added).

3.2.4 Duals of Duality

The above clarity leads to the next question: what are the *duals* in logical duality? That is, duality is present between what type of things? It might appear initially that the duality is amidst logical schemata. However, as discussed, there are multiple operations through which the dual of a schema can be formulated and even though the resultant schemata appear different, they are logically equivalent. This suggests, alternative to the initial presumption, that duality might be between two truth-tables, each of which can be symbolic represented in more than one ways. Thus, the equivalence among the different schemata that are the duals for a schema is a good reason for considering that truth-tables and not schemata as duals in logical duality.

Apart from the above reason, there is a much more stronger reason to consider that duality is between truth-tables and this again draws from equivalence among schemata. Generally speaking, schemata are equivalent if they agree with each other for similar truth-value analysis. This implies that a schema p is equivalent to schemata likes $p \wedge (p \vee q)$ and $(p \wedge q) \vee (p\wedge \sim q)$ since they all behave in the same way under similar truth-value analysis (ibid., 47). But, all these schemata mentioned as equivalent have different truth-tables. And this implies that the dual of each of these schemata will be different even though logically they are equivalent. Therefore, two logical schemata might have different duals even though they are equivalent. This confirms my suggestion that duality discussed above is amidst truth-tables. One of the main reason why equivalence among schemata does not guarantee that their duals will also be equivalent is because equivalence can be maintained across schemata that have different number of components. Contrast to this, truth-table's equivalence and duality require same number of rows and columns.

3.3 Duality in Projective Geometry

3.3.1 Duality of Points and Lines

In projective geometry, axioms, implied theorems and configurations (i.e., the geometric figures) are articulated largely through primitive concepts like lines, points, incidence and separation (Coxeter 1993, 12). In this context, a particular kind of duality arises due to the exchange of the terms 'points' and 'lines' (along with the accompanying terms like collinear, coincidence, etc.) in the statements found here. To illustrate this, consider a simple axiom of projective geometry: "any two distinct points are incident with just one line". In this, the interchange of terms 'lines' and 'points' results in a statement — "any two lines are incident with just one point". Now, this new statement can also be considered as one of the axioms for further elucidating and verifying the theorems and statements of projective geometry (Coxeter 1987, 25). It can be observed that these two statements, which are considered as duals of each other, are completely different: one is about collinearity and the other is on the property of coincidence respectively. Similar to this illustration, other axioms of projective geometry can be respectively *dualized*. Therefore, for every theorem found here, a corresponding dual theorem can be framed. For instance, consider the *Desargues' theorem*: "If two triangles are perspective from a point they are perspective from a line". Dualising the theorem, would give its dual theorem: "If two triangles are perspective from a line they are perspective from a point" (ibid., 19). The duality of projective geometry is not limited to theorems alone; it is observed among geometric figures as well (Casse 2006, 19). To provide a simple illustration of "dual configurations", consider the figures quadrilateral and quadrangle. Quadrilateral is a system of four lines coinciding at six distinct points. In contrast to that, quadrangle is a system of 4 points that are joined by six lines (Coxeter 1987, 7). Important aspects of quadrangle's description (like, e.g., opposite sides and diagonal point) gets inverted in the case of quadrilateral (opposite vertices and diagonal line). Of course, "inversion" here refers to the interchange of points and lines in the definition of important aspects (like 'diagonal' or 'opposite') of both the figures.

3.3.2 Principle of Duality

As the above examples highlight, a statement (about an axiom, a theorem or a configuration) in projective geometry yields another statement that is still meaningful. This characteristic aspect of the statements is captured by the *principle of duality*, which states that "every definition remains significant, and every theorem remains true, when we interchange the words *point* and *line* (and consequently also certain other pairs of words such as join and meet, collinear and concurrent, vertex and side and so forth)" (ibid., 25). This is the principle in two dimensional space. In three dimensional projective geometry, similar duality is present between lines and planes. In the principle of duality, the phrase "remains significance . . . remains true" means that the dual of an axiom is still an axiom and the dual of a configuration yields a meaningful configuration. In the case of theorems especially, dualizing a theorem yields another one which is again valid. It is because of this, historically, dualizing process has been considered as a legitimate justification principle to validate theorems. Joseph Diez Gergonne, a nineteenth century mathematician, recognized this aspect about dual theorems and considered duality as a "universal" principle (Boyer 1956, 239). For him, an already verified theorem couched in one term, can be converted to a new theorem through the interchange of the terms "line" and "point" and this resultant theorem is valid due to the principle of duality.

This characteristic possibility — of swapping the terms — is observed in the projective geometry alone because points and lines are related fundamentally: any two lines always have a unique incidence point. Since duality prevails due the interchange of the terms in the statements, similar to what was observed in the case of logical schemata, dualizing procedure here too can be articulated symbolically. To illustrate this, consider that points are represented by uppercase letters (A,B,C ...), lines by lowercase letters (a,b,c, ...) and incidence by dots. Given this, $AB \cdot CD$ indicates a point where the line between the points A and B intersect the other line between C and D. Similarly, a line can be represented as $(a \cdot b)(c \cdot d)$, where $(a \cdot b)$ is the point where the two lines a and b intersect. With this scheme of representation, dualizing can be understood as a symbolic operation that involves not only "interchange of capital and small letters but also the removal of any dots that are present and the insertion of dots where they are absent" (Coxeter 1987, 25).

Since the dualizing procedure involves exchange of terms in the statements, are there any instances where exchange of the terms in a statement yields the same statement? Cases like this would qualify as *self-duals* and in projective geometry, there are several instances of this. Among configurations, the system constituted by three points and three lines, which is nothing but a triangle, is one such instance. Here, the description "a family of three non-collinear points and three lines joining them" and its dual "a family of three non-concurrent lines and three points where they meet" both describe the same configuration (Brannan et al. 2011, 176). Self-duality is also observed among certain transformations found in projective geometry. Projective correlation is the correspondence between points and lines in a plane such that in this transformation the relation of incidence is preserved (Coxeter 1987, 57). This correlation therefore transforms lines to points in a plane and vice versa. Here, if the line-to-point transformation is represented as $x \to X'$, then the dual of this transformation (where the points and lines are interchanged) — $X \to x'$ — is again a projective correlation. Hence, projective correlation is considered as a self-dual concept.

To arrive at the dual of a given axiom or theorem, the procedure that needs to be followed seems quite straightforward — exchange the terms "line" and "point". This operation is similar in nature to the dualizing procedure observed in the case of logical schemata. Regarding this duality operation, there have been several interpretations. Difference in opinion about the actual nature of this dualizing operation started in the nineteenth century, amidst the conflict between two different approaches of doing geometry: analytic and synthetic methods. During this time, the concept of duality became a prominent topic of discussion due to two geometricians: Jean-Victor Poncelet, who promoted synthetic geometry, and Joseph Gergonne, who practised analytic method. Each of these geometricians had different interpretation of the dualizing operation. Gergonne held that the *principle of duality* is a unique procedure in projective geometry and it is the symmetrical nature of the axioms of incidence which justifies this interchange of terms (Coxeter 1993, 16). Opposed to this, Poncelet held that this exchange of terms is nothing but the *principle of reciprocal polars* (Boyer 1956, 239). According to this principle, every point (which is called the polar) can be mapped to a unique line (called the pole) with respect to a conic and vice versa. Since there is a correlation between a point and a line, Poncelet argued that this superficial exchange of terms is in fact switching between pole and polars. Therefore, dual theorems or configurations arrived through the interchange of terms are nothing more than polar reciprocals of original theorems or configurations.

In spite of some similarities, as Pedoe (1975) discusses clearly, the principle of reciprocal polars (R) is different from the principle of duality (D). One of the first difference between these two principles was provided by Gergonne himself (Boyer 1956, 239). R is always articulated in the context of a conic. Contrast to this, the procedure specified in D does not depend on one. Because of this characteristic, D was considered as a "general property of the space" (Pedoe 1975, 277). The above difference already hints that R is more specific than D. This characteristic difference is clearly brought out in the case of specific configurations in projective geometry — Desargues configuration and Pappus configuration. When evaluated under P, both these configurations turn out to be self-duals (i.e., each configuration is its own dual after the interchange of terms). However, in the context of R, only Desargues configuration is self-polar. Pappus configuration does not exhibit self-polarity (ibid., 276). Because of this, Coxeter comments that "the old controversy between Poncelet and Gergonne is settled in the latter's favour" (Coxeter 1993, 70). Finally, certain theorems specifically related to conics can be transformed only by R; in this regard, D is ineffective to bring these transformations. For instance, theorems of circle can be transformed into theorems about conics and vice versa using R (Pedoe 1975, 276). These points clearly show that the process of interchanging 'point' and 'line' in theorems is a different process compared to the operation of switching the reciprocals. Drawing the distinction between these two principles settles the historical argument between Poncelet and Gergonne. Principle of duality is indeed unique and does not require principle of reciprocal polars for its justification (Nagel 1939, 185).

3.3.3 Duality and Formal Geometry

The above discussion clearly highlights that the principle of duality brings forth the relation between points and lines. And discussing this relation is crucial for understanding the nature of duality found in projective geometry. To describe this, a useful place to begin is the axioms of incidence. The first column of Table 3.6 lists some axioms of incidence. The second column lists the duals of these axioms arrived through the interchange of terms. The final column lists the same axioms phrased such that they already imply their duals.

Therefore, the axioms in the last columns, unlike the other counterparts in the previous two columns, are self-dual statements. The possibility of phrasing the axioms

Axioms	Dual axioms	Self-dual axioms
Any two distinct points are incident with just one line	Any two distinct lines are incident with just one point	Two distinct points cannot both be incident with two distinct lines
Any two lines are incident with at least one point	Any two points are incident with at least one line	There exist two points and two lines such that each of the points is incident with just one of the lines

Table 3.6: Axioms of incidence; taken from Coxeter (1993, 14-16).

in this way brings to the forefront the central feature of these axioms that grounds the interchange of terms. Gergonne was first to recognize this symmetrical nature of the axioms (Boyer 1956, 239).[4] Here, the "symmetry" of the axioms refers to the invariance of these axioms under the exchange of the terms (Coxeter 1993, 16). Apart from these axioms, mathematical expressions of lines and points in two dimensional projective geometry exhibit this symmetry. To illustrate this, consider a simple example. In projective geometry, a line $[u, v, w]$ is represented by the equation $ux + vy + wz = 0$, where (x, y, z) are the homogeneous coordinates of the points in the space. However, the same expression $ux+vy+wz = 0$ can also be interpreted as representing a point (x, y, z) in a space defined by the homogeneous coordinates of lines represented by $[u, v, w]$. Therefore, the above expression exhibits similar invariance as seen in the case of the axioms. Given this, Julius Plücker, a nineteenth century mathematician, suggested that the principle of duality is simply a consequence of this symmetry found in the incidence relation of points and lines (Boyer 1956, 250). In fact, Plücker further showed that this "duality" has nothing specifically to do with *points* and *lines*; rather it is due to the same dimensionality that these two entities share. Since the representation of projective space or plane depends on the dimensionality of elements chosen as fundamental, Plücker showed that duality can be seen "between any pairs of configurations whose dimension-numbers . . . with respect to the manifolds in which they occur is the same" (Nagel 1939, 191). Therefore, the duality in projective geometry is not something specific about points and lines since similar invariance is observed if fundamental entities of this geometry are lines and planes or other pair of entities. Therefore, this duality has to do *generally* with the pair of objects of this geometry, whatever they might be. The realization that this duality pertains to the relation among entities of the geometry without any emphasis on the exact kinds of entities involved indicates that the source of this duality lies in the structure of the geometry.

4. Gergonne was not consistent with the explanations he provided for the validity of the principle of duality. Apart from this analysis, he also suggested that the principle of duality is based on the "nature of extension" itself (Nagel 1939, 182).

This is the right place to discuss the role of this duality principle in the rise of formal geometry. Geometry underwent a major revision in the nineteenth century. Classical geometry was understood as a quantitative science that studied metrical relations and had, as its subject matter, space or extension (Nagel 1939, 143). In the nineteenth century, these fundamental aspects were questioned, largely due to the influence of algebra. With this, several important changes happened in geometry. First, non-metric, non-visualisable entities, like "imaginary" and "ideal" points, were introduced as legitimate objects (ibid., 150). Along with that, points as the only type of fundamental entities of geometry was questioned. This led to the possibility of having other kinds of entities (e.g., lines and planes) as the "absolute simples" for a geometry (ibid., 179).[5] This gradual shift led to dethroning of objects (like points, lines, and planes) as the subject-matter of geometry. In contrast to this, with the emphasis on relations between entities, geometry became the discipline which studies "the interconnections of the symbolic operations whose properties have been formally specified" (ibid., 178). Thus, post nineteenth century, abstract structures and not particular things, became the subject matter of geometry (Torretti 1978, 190).

The brief historical account of the development of formal geometry is important to understand the interpretation of duality in projective geometry. In fact, the principle of duality in projective geometry played a vital role in the raise of structuralism in geometry. First, in the homogeneous coordinate system, the dual interpretation of $ux + vy + wz = 0$ provided the grounds for realizing that there can be more than one fundamental entities of a geometry (Nagel 1939, 187). More importantly, the generalization of this duality across different pairs of entities provided the motivation for interpreting the representation of entities (like (x, y, z) or $[u, v, w]$) in expressions as mere place-holders that can take any value. These fundamental entities became "arbitrarily selected marks, whose sole function is to serve as 'places' or blanks to be filled in as occasion may warrant. The result of such a formalization is an 'empty frame', which expresses the structure of the set of nuclear propositions and which is alone relevant for pure geometry" (ibid., 197).

The above historical discussion provides context for analysing the difference between R and D, which were mentioned earlier. By recognising that duality is not due to specific relation (reciprocity) between entities (pole and polar), but something more general than this, the structural interpretation was anticipated. The centrality of the principle of duality in the formalism of geometry provides motivation to look for the structural interpretation of duality. In order to attempt that, consider Desargues's theorem and its dual given in Table 3.7 as a working example of duality. These two statements of projective geometry —

5. There is an interesting historical note about duality and objects. Michel Chasles, a French mathematician, observed that there can be two kinds of geometries, one with points as the fundamental entities and the other which had lines as its fundamental entities. With this, he proposed "universal dualism" according which every discipline has this duality of two fundamental entities. He observed that mechanics, at that point, has only one object of study — bodies; and suggested that there might be a new science which has a different object of study (Nagel 1939, 187)

Desargues' theorem (s_1)	**Dual theorem** (s_2)
Let two triangles be such that the lines joining corresponding vertices meet at a point. Then the points of intersection of the corresponding sides of the two triangles are collinear	Let two triangles be such that the points through which corresponding sides pass are collinear. Then the lines through the corresponding vertices of the two triangles are concurrent.

Table 3.7: Desargues' theorem and its dual theorem. Taken from (Brannan et al. 2011, 178)

s_1 and s_2 — are dual to each other since all the terms related to *point* (to be referred as 'point-terms') and terms related to *lines* (to be referred as 'line-terms') in one sentence undergo a *thoroughgoing interchange* — to use a phrase that has played a prominent role in logical duality — to result in the other statement. Now, taking suggestion from the above discussion, where I had remarked that the entity referent terms become just "blanks to be filled" due to formalization in projective geometry, the point and line terms in s_1 and s_2 will be replaced by "empty frames" in order to arrive at the skeletal structure of the underlying statement. This would result in the following statement

S : Let two triangles be such that the $[O_1]$ ... corresponding $[O_2\, term]$ ... $[O_1\, term]$. Then the $[O_2]$ of the corresponding $[O_1\, term]$ of the two triangles are $[O_2\, term]$

By rephrasing in this fashion, it might appear that, I still have not done anything interesting as S does not seem to be that different from s_1 or s_2. Similar to 'O_1' or 'O_2', 'line' or 'point' are again general labels in these statements. However, this rephrasing generalizes s_1 and s_2 by identifying the two kinds of place-holders present in them. With this, I can now represent s_1 as $S[lines, points]$ and s_2 as $S[points, lines]$, where the inputs to S indicate the values for O_1 and O_2 terms respectively. Thus, S clearly exhibit the structural similarity of dual statements. And it is in the backdrop of this similarity, the difference between the statements s_1 and s_2 can be stated: they differ only in the *order* of the inputs to $S[O_1, O_2]$. Therefore, the duality between s_1 and s_2 is due to the *reversal* of the order, which can be seen clearly when represented like: $S[lines, points]$ and $S[points, lines]$.

The above discussion provides the initial acquaintance of the structural interpretation of duality. This kind of duality is generally found when there is a class of entities (O_1, O_2, S), where O_1 and O_2 are objects (like lines, planes and points) and S are the statements about O_1 and O_2. Of course, this class has its specific rules and features because of which duality is possible. For instance, duality like this is not observed in Euclidean geometry since in it, two parallel lines never have an incident point (projective geometry assigns an incident point for parallel lines too). Given that duality is the reversal in the order of the inputs to statements, the process of flipping the inputs results in two different statements. Hence, duality is quite opposite of commutative relation. A relation between two objects is commutative when the reversal of the relata does not change the statement. For instance, relations like ‘spouse of’ is commutative and ‘parent of’ is not commutative. What seems to be central to this duality, therefore, is some form of *inverse*. This reversal of objects can be performed in the background of a structure that makes this inversion possible. In other words, projective duality is the *inverse* of inputs with the entire structure remaining same. Here, it should be noted that populating a given structure differently does not produce duality. There has to be some reversal involved. Structural similarity by itself does not constitute the duality in projective geometry; the reversal (of the inputs) is also a crucial aspect of it. This is where I disagree with Nagel (1939), for whom structural similarity between two statements is sufficient to invoke the notion of duality. Let me provide an instance where Nagel reduces duality to just structural similarity. While discussing the developments in geometry due to Felix Klein’s contribution, Nagel shows how two different geometries can be structurally similar to each other (i.e., have similar transformations) even though each of them have different objects of study. Here, while comparing two different manifolds of elements A and A', he says “for every theorem about the invariant properties of A there will correspond a dual theorem about the invariant properties of A', and conversely” (ibid., 206). For Nagel, therefore, two theorems which are about different entities but are structurally similar constitute a dual theorems. However, as I have shown above, this context does not demand invoking duality as there is no reversal of entities involved.

3.4 Duality of Vector Spaces

3.4.1 Dual and Second Dual Spaces

Functional analysis, which is a prominent branch of abstract algebra, is another place where duality is invoked. Here, duality is observed between vector spaces and the linear functionals described over them. Functional analysis deals with operations prescribed over abstract spaces. As the “abstract” in the name indicates, an abstract space consists of a set of abstract elements (“whose nature is left unspecified”) that possess some structure (like distance function) (Kreyszig 1978, 3). Therefore, what differentiates an abstract space from a set is the presence of some structure among its elements. This kind of general definition of space provides the flexibility of accommodating not only the obvious kinds — Euclidean, complex space — but also other sets of entities as “spaces” like real line, complex plane, function space, etc. Similarly, a set of vectors having some structural relation between them constitute a “vector space”. Here, the structure is provided by two algebraic operations: addition of vectors and multiplication of vectors by scalars (ibid., 50). Another important term which needs to introduced is *functionals*. Abstract spaces are related to one another through operators that simply map an element of one space to an element of another space (ibid., 82). Functionals are specific kind of operators that map an element of a vector space to a scalar entity. Therefore, the domain of functionals are vector spaces and range is either real line or complex plane (Kreyszig 1978, 103). Given vector spaces and functionals defined over them, the duality found amidst these spaces can be articulated. As mentioned, in the abstract interpretation of spaces, any set of entities having some structure can be considered as a space. In this way of thinking, it can be shown that the set of linear functionals defined over a vector space itself forms a vector space (ibid., 106). If X is a vector space, then the functionals — f_1, f_2, …— defined over this space constitute a space. This implies that the two operations mentioned above — addition of entities and multiplication of entities by scalar values — can be defined meaningfully over these functionals too. Therefore, for a vector space X, there is an accompanying vector space X' constituted by all functionals, each of which map the elements of X to either real line or complex plane. This X' is called the algebraic *dual* space of X (ibid., 106).

The above discussion provides the way of formulating a dual space X' (also known as the conjugate space) for a given vector space X. Given that X' is a space in itself, it seems legitimate to look for the dual of this space. A set of linear functionals g can be defined over X' such that they map every f in X' to a scalar. The space, $(X')'$ or written simply X'', which this set would constitute is called the *second dual* space of X. In order to understand how spaces X, the dual X' and the X'' are related to each other, consider linear functionals g_1, g_2, …, etc., of X'' analogous to how the set of linear functionals of X' relate to X. As mentioned earlier, f generally refers to all linear functionals of X' that can be described over the vectors x in X. Therefore, $f(x)$ generally represents a function that maps every $x \in X$ to a scalar entity. Parallel to this, the functionals g of X'' can be understood as $g(f)$ implying that these are the functionals that take f as their variables. This formulation clearly suggests that $g(f)$ behaves similar to $f(x)$. $f(x)$ is a general representation of various functionals like $f_1(x)$, $f_2(x)$, etc. Here, $f_i(x)$ is a specific function which provides the range of scalars for x that varies over the domain of X. Similarly, $g(f)$ can be understood as the general representation of $g_1(f)$, $g_2(f)$, etc., and $g_i(f)$ provides the domain of functions f_i, f_j, etc., in X' and map them to specific scalars. This mapping of various functionals f to scalars by g suggest that there is a correspondence between g in X'' and x in X. This is because, $g_i(f)$ can be interpreted as selecting a series of functionals f for a specific x in X. Therefore, there is a one to one mapping, which is called the *canonical mapping*, between every $x \in X$ and a specific g_x in X'' (ibid., 108). With this mapping between X'' and X, the relation between X, X' and X'' can now be succinctly articulated by writing the pair of general entities x and f involved here as $[x, f]$. Using this representation, $f(x)$ is understood as varying x for a specific f. Parallel to this, since there is a mapping between g_x and x, $g(f)$ can be understood as varying f in $[x, f]$ by holding on to the same x (ibid., 107). However, the canonical mapping between X and X'' is not true for all kinds of vector spaces. Whenever the canonical mapping is present, it can be shown that X'' is *isomorphic* to X (Kreyszig 1978, 108). Isomorphism between two spaces indicate that these two spaces have the same structure among their elements. If the the spaces are isomorphic and canonical mapping is possible, then the given vector space X is considered to be *reflexive* (ibid., 109).

To illustrate the above discussion in a specific context, consider the duality of finite dimensional normed spaces. The main motivation for considering finite dimensional spaces is that all these spaces are *reflexive*.[6] Hence, the dual of dual spaces of finite dimensional vector spaces will be isomorphic to them. As already mentioned, for a particular set to constitute a vector space, there should be some structure among its elements. Along with these, if there is another real-valued function called *norm* that can be defined, then this set constitutes a normed vector space. Norm, which has specific characteristics (e.g., it is non-zero for every element), essentially helps in defining the metric for the space. Given this, the set of all linear functionals on the given finite dimensional normed space X constitute a dual space X', which also has a specific norm (ibid., 120). Here, it should be noted that the norm of the dual spaces are different. However, the second dual space X'' got through X' will have the same norm as X. Of course, this is because X'' is reflexive and isomorphic to X (ibid., 240).

The context of the above discussion also makes it possible to introduce the notion of self-duality of vector space. A space X is dual to itself when when X' turns out to be isomorphic to X. The real coordinate space of n dimensions R^n is an example. It can be shown that $R^{n\prime}$ has the same norm as R^n and hence isomorphic to it (ibid., 121).

3.4.2 Duality of Objects or Structures?

From the above discussion, it seems that the duality found here is amidst distinct vector spaces. However, if duality is interpreted along this line, then, as I will show, it is difficult to make sense of duals and the dualizing operation. The space of linear functionals defined on a given vector space X is straightforwardly labelled as the *dual* of X. This appellation is specifically for X' and it is not reciprocally applicable to X as well. That is, X is not usually called as the dual of X'. In fact, when X is reflexive, it is considered isomorphic to the *second dual* space X''. Therefore, it should be noted that the usage of “dual” does not have the similar connotation observed in, for instance, logic and projective geometry, where “duals” referred to always a pair of entities and “dual” was equally applicable to each of them. Here, in the case of vector spaces, clearly there seems to be a “parent” space out of which the “dual” space is arrived at. Hence, there seems to be only *dual* spaces (spaces that are labelled “dual”) and no *dual spaces* (a pair of spaces that are dual to each other). This clarification points at the shortcomings of labelling the spaces as “duals” and presuming that “duality” exist between distinct spaces. Similar problem arises in

6. Apart from finite dimensional spaces, even Hilbert spaces are reflexive.

the interpretation of the dualizing operation. In other dualities observed prior to this, there is a specific operation which is carried out on the entity (logical schemata or a theorem of projective geometry) in order to arrive at the dual. This dualizing operation was *involutory* such that when this was carried on the dual, the original object is recovered. In vector spaces, the dualizing procedure involves defining another space for a given X. This resultant space X' is constituted by defining a particular kind of mapping (linear functionals) on the elements of a given space. Thus, this procedure of constituting a space by defining a set of linear functionals is equally meaningful and applicable to X'. However, this operation is not straightforwardly involutory: the application of this procedure twice, does not always give back the initial object.[7] Moreover, this operation of defining the space does not directly involve reversal of some elements, like the swapping of truth-values or "lines" with "points".

If duality is understood not between distinct spaces, but between two structurally distinct spaces, then the above discussed shortcomings of duality do not arise. Consider the two spaces involved in this duality: X and X'. Both these spaces are of the same kind — they are vector spaces. But this consideration of spaces is independent of the elements constituting the space. In this sense, these spaces are abstract and are defined only by their structure (the operations and the norm). So, even though the dualizing procedure starts off with defining a space of linear functionals on a space of vectors, this consideration of the kinds of entities constituting each of these spaces is not relevant. It follows then that duality pertains to the structural features of X and X'. The centrality of the structure of spaces for duality becomes evident in the discussion of the second dual space X'' and the reflexivity of X. As mentioned earlier, if X is reflexive, then the dual of X' is isomorphic to X. Since, structure characterizes an abstract space, all that can be meaningfully said is that these two spaces X and X'' are isomorphic. Hence, there is no way to consider that X is distinct from X''. In this structural interpretation of spaces, then, the previously mentioned confusions of understanding this duality through the dual operation and usage of 'dual' in vector spaces get clarified. Given a particular space X, there is X' that is structurally dual such that the same dualizing procedure when carried out on X' gives X back. According to this understanding, there are always dual spaces: given a space, irrespective of whether it is a space of linear functionals or vector space, there is a corresponding dual. And there is a way of defining certain kind of mapping (through linear functionals) on this space such that these mappings form its dual. This also implies that there is conceptual priority between X or X'. Of course, this neat interpretation of duality is possible only among certain kinds of spaces.

However, when spaces are characterized only by their entities, there is another aspect about duality that becomes to difficult characterize. Consider that two spaces X and X' are finite normed spaces which are dual to each other. When a certain kind of mapping

7. As mentioned, only for reflexive spaces, this is possible.

over X is defined, then we end up with a space X'. And this is true the other way round as well. Therefore, duality seems to be nothing more than certain kind of functionals on X constituting another space X'. It can be immediately seen that in this interpretation, duality conceptually signifies the toggling between two spaces X and X'. In logic and projective geometry, duality stood for some kind of inversion or reversal of specific attributes while keeping other aspects in tact. For instance, in projective geometry, the order of objects were reversed by keeping the structural relations in tact. However, in vector spaces, duality understood as two spaces coupled to each other, there is no reversal or inversion involved. Given this lack of inversion, should not be X and X' be considered as *polar* spaces? The suggestion to consider these coupled spaces as "polar", instead of dual, spaces arises due to historical reasons. Hans Hahn is credited with the conception of dual spaces in functional analysis. While working on the "extension problem" — where the task is to extend a mathematical object (like functionals) defined over a subset to the entire set — he conceptualized the *polare raum* to show the applicability of his theorem (which is currently called the Hahn-Banach theorem) (Dieudonné 1981, 137). Therefore, in the context of its conception, the X' space was labelled (in German language) as the "polar space". Given that in the present discussion too there is no reversal observed, it seems better to label the spaces X and X' as "polar spaces" rather than "dual spaces". This also implies that there is no duality as such among these spaces.

However, there might be a particular way through which the above dismissal of duality among spaces can be addressed. In the above discussion, it was observed that spaces X and X' are indeed structurally related. But mere structural relation does not justify invoking these two spaces as duals; what more is required is the presence of some form of inversion or reversal. Here, I will provide one way of addressing this lacunae. Consider X' is a space constituted by functionals $f(x)$ and X'', which is isomorphic to X, is a space constituted by $g(x)$. Given this, the difference between $f(x)$ and $g(x)$ is the following. If a mapping is defined schematically as $[fixed, variable]$, then $f(x)$ can be represented as $[f, x]$ indicating that a given function varies over a given range of X. Compared to this, $g(x)$ gets the form $[x, f]$ implying that for a given x, f is varied. In the schematic way of articulating this scenario, the reversal between $f(x)$ and $g(x)$ can be clearly seen. And given a particular space of x, there is only a corresponding set of functionals $f(x)$ that can be described. With this, these two spaces can be considered as *duals* of each other.

3.5 Duality of Groups

In group theory, duality is formulated amidst theorems that pertain to homomorphisms between groups. *Groups* are basically sets of certain elements $a, b, \ldots$ and a binary operation $*$ such that the following axioms are respected:

Axiom of identity For the given operation $*$, there is an identity element e such that $a * e = e * a = a$

Axiom of closure The product of two elements under the operation $a * b = c$ also belongs to the set

Axiom of invertibility For every element a under the operation, there is an inverse element b such that $a * b = e$

Axiom of associativity The operation is such that $(a * b) * c = a * (b * c)$

Homomorphisms are functions that can be defined between two groups such that these functions preserve the groups' structure in a specific sense. To illustrate the duality in group theory, consider the example provided by Saunders MacLane (1950). MacLane first states the following two theorems of group theory and proves them individually.

Theorem 1 The abelian group F is *free* if and only if, whenever $\rho : B \to A$ is a homomorphism of an abelian group B onto an abelian group A, and $\alpha : F \to A$ is a homomorphism of F into A, there exists a homomorphism $\beta : F \to B$ with $\rho\beta = \alpha$

$$\begin{tikzcd} F \arrow[r, "\beta"] \arrow[dr, "\alpha"'] & B \arrow[d, "\rho"] \\ & A \end{tikzcd}$$

Figure 3.1: Theorem 1

Theorem 2 The abelian group D is *infinitely divisible* if and only if whenever $\lambda : A \to B$ is an isomorphism of an abelian group A into an abelian group B and $\alpha : A \to D$ a homomorphism of A into D, there exists a homomorphism $\beta : B \to D$ with $\beta\lambda = \alpha$

$$\begin{tikzcd} D & B \arrow[l, "\beta"'] \\ & A \arrow[ul, "\alpha"] \arrow[u, "\lambda"'] \end{tikzcd}$$

Figure 3.2: Theorem 2

The above two theorems, MacLane indicates, can be considered as "dual" to one another since one theorem under a transformation process becomes the other theorem. This transformation involves two steps. First the terms in a theorem (e.g., homomorphism like α) needs to be swapped by other terms (e.g., isomorphism like λ). Second, the direction of the functions (the arrows in the figures) and the order of the factors in the products have to be reversed (MacLane 1950, 486). Since both theorems transform to one another in the above sense, MacLane states that free abelian groups and infinitely divisible abelian groups are dual to one another. With this illustration, MacLane proposes the general strategy to "dualise" a theorem in group theory: for any statement about groups that refers only to homomorphisms, quotient groups and projection, a corresponding dual statement can be formed by interchanging the existing terms with isomorphisms, subgroups and injections respectively along with changing the order of the products involved (ibid., 488). MacLane demonstrates how this procedure can be used to formulate dual theorems in several other cases. For instance, a theorem about *direct product* about groups can be shown to be dual to a theorem about *free product* of groups (ibid., 490). Similarly, theorems about ascending and descending central series of groups can be shown to be dual to one another (ibid., 493). Since the duality in group theory is articulated amidst statements about specific group concepts and operations, the theorems of group theory are the duals. In this sense, the duality found in group theory is similar to the one found in projective geometry.

3.6 Duality of Categories

A *category* is a mathematical object which consists of certain objects $A, B, C, \ldots$ and maps $f, g, h \ldots$, which are also called as "arrows", between the objects of the category (Awodey 2010, 4-5). These arrows satisfy the following conditions:

- for every object A, there is an *identity* arrow such that $1_A : A \to A$
- if $f : A \to B$ and $g : B \to C$, then a *composite* arrow can be defined such that $g * f = A \to C$
- the composite arrows respect associativity such that $f * (g * h) = (f * g) * h$

The notion of duality in category theory is articulated in the context of equivalence of categories. In order to introduce the notion of equivalence, few basic concepts needs to be defined first. In category theory, homomorphisms among categories are called *functors*. Since categories consist of not only objects but also arrows between them, a functor between two categories $F : \boldsymbol{C} \to \boldsymbol{D}$ map both the objects and the arrows of $\boldsymbol{C}$ to that of $\boldsymbol{D}$ (ibid., 8). Given these functors $F, G, \ldots$ between two categories, a further notion of maps, called as *natural transformations*, amidst these functors can be defined (Leinster 2014, 36). Defining these maps allows for formulating *functor category* $[\boldsymbol{C}, \boldsymbol{D}]$ consisting of functors and the maps between these functors (ibid., 30). In this category $[\boldsymbol{C}, \boldsymbol{D}]$, there are few maps that are isomorphisms and these are labelled as *natural isomorphisms*. With the help of these concepts, different notions of equality for categories can be defined. The basic notion of equality between categories implies that the objects and the maps of both the categories are the same. Since the criterion is "unreasonably strict", this notion of equality is not suitable for categories (Leinster 2014, 33). A more interesting and useful notion of equality can be defined using natural isomorphisms. Two categories $\boldsymbol{C}$ and $\boldsymbol{D}$ are *isomorphic* if the functors between them $F : \boldsymbol{C} \to \boldsymbol{D}$ and $G : \boldsymbol{D} \to \boldsymbol{C}$ are such that

$$F * G = 1_D \quad G * F = 1_C$$

where 1_D and 1_C are identity functors of the two respective categories (Awodey 2010, 172). However, $\boldsymbol{C} \cong \boldsymbol{D}$, where $\cong$ represents isomorphism, is still "too strict" for categories (Leinster 2014, 33). Because of this, a much more relaxed notion called *equivalence* is defined for categories. Two categories are *equivalent*, represented as $\boldsymbol{C} \simeq \boldsymbol{D}$, when the two functors F and G are related such that (Awodey 2010, 172):

$$F * G \ \cong \ 1_D \quad G * F \ \cong \ 1_C.$$

As can be seen, unlike the scenario of $\boldsymbol{C} \cong \boldsymbol{D}$, where the functors were equivalent to the identity functors, in $\boldsymbol{C} \simeq \boldsymbol{D}$ the functors are just isomorphic to the identity functors. This kind of equality "returns the original thing only up to isomorphism, not 'on the nose' " (ibid., 178). With equivalence amidst categories understood, duality in category theory can be defined. An equivalence of the form $\boldsymbol{C} \simeq \boldsymbol{D}^{op}$ is considered as *duality* between these two categories (Leinster 2014, 35). Here, $\boldsymbol{D}^{op}$ represents an opposite category of $\boldsymbol{D}$ because it has all the elements of $\boldsymbol{D}$ but with arrows reversed (ibid., 16). A concrete example of this duality is illustrated by the *Stone duality* theorem, according to which "the category of finite boolean algebras is equivalent to the opposite of the category of finite sets" (Awodey 2010, 178).

3.7 Dual Theories of Physics

In certain areas of modern physics, especially in the domains of quantum field theories (QFT) and string theories, there are theories which are said to be *dual* of each other. This kind of duality consists of two theories that are related to each other in several ways. First of all, these two theories are about the same phenomenon and are *theoretically equivalent*, which briefly means that there is no difference in physical accounts of phenomena or system provided by these two theories. But being theoretically equivalent does not imply that they are the same theory through and through. Even though they describe a given phenomenon or system similarly, there might be differences in the way they achieve this. Apart from that, these two theories might differ about the objects they deal with, the range within which these theories give appropriate values, etc. Amidst these differences, if they are dual theories, there is a particular mapping that relates the differences between these two theories. It is the presence of this mapping that is the central characteristic of this duality.

The following example illustrates the above duality. Consider that there is a system whose Hamiltonian can be articulated using two fields — ϕ and ϕ' — that are related in a way which I will specify below. The perturbation series of this system expressed in terms of these fields respectively will be

$$H = H_0 + gH_1 = H_0{}' + g'H_1{}'$$

In these expressions, g and g' are the coupling factors and these are related to each other through the relation $g = 1/g'$. Because of this reciprocal relation between these two ways of talking about the system, there are two alternative ways of studying the same system. Given that both these approaches are equivalent descriptions of the system, when g is very large, then the system can be analysed through g' given the perturbation series would be more accurate in this form (Polchinski 2017, 7). Because of the reciprocal relationship between g and g', the representation of the system through fields ϕ and ϕ' will have reciprocal relation. If using one of the fields, say ϕ, provides quantum fluctuations at large values of g, then shifting to the representation of the system using ϕ' will provide classical behaviour at lower values of g. Therefore, the presence of two theories for the same phenomenon and these two theories being related through an "equivalence map", where the weak coupling regime of one theory is equivalent to the strong coupling regime of the other theory, is being called as duality here (Castellani 2017, 100). This kind of duality encountered in quantum field theory and string theory is labelled as *strong/weak duality* and also *S-duality*, where 'S' stands for the symmetry group $SL(2, Z)$ (ibid., 100).

To illustrate one instance of S-duality, consider the duality between electric and magnetic fields. The relation between electric $\overrightarrow{E}$ and magnetic fields $\overrightarrow{B}$, which was recognized during the classical times itself, became even more evident with Maxwell's theory. Through this, it was easy to observe that both these fields are equivalent to one another in a specific sense since one of them can be transformed into the other through transformations like $\overrightarrow{E} \rightarrow \overrightarrow{B}$ and $\overrightarrow{B} \rightarrow -\overrightarrow{E}$ (ibid., 101). This dual transformation between $\overrightarrow{E}$ and $\overrightarrow{B}$ implies that both interpretations of a field — either as magnetic or as electric — are equivalent descriptions. This equivalence suggests that there is one to one correspondence between sources and charges of the fields. Because of this duality, the equivalent of electric charge (q) for magnetic field — "magnetic charge" (g) — was stipulated. Dirac, in 1931, by demonstrating this equivalence within quantum framework, provided the quantization condition for fields, where it was shown that g and q are related to each other as $qg = 2\pi n$ where $n = 0, \pm 1, \pm 2, \dots$ (ibid., 101). As this relation shows, the charges are inversely related to one another.

The complete equivalence between electric and magnetic fields in QFT was demonstrated in 1970s. This was possible to achieve through other subsequent accomplishments. To begin with, the equivalence between sine-Gordon theory and massive Thirring model, theories about two dimensional fields, was shown by Coleman and Mandelstam in 1975. Specifically, it was shown that states of solitons in sine-Gordon theory were equivalent to elementary particles' states described by massive Thirring model. The coupling constants of the fields given by each of these theories exhibited equivalence that is usually termed as "weak/strong duality" (Castellani 2017, 103). The equivalence gets this label because one theory's coupling constant in its weak regime corresponds to the other theory's coupling constant in the strong regime. This equivalence was further generalised for four dimensional field theory in 1977 by Montonen and Olive, who showed evidence for dual symmetry between two formulations of the same theory such that magnetic and electric charges swap under this transformation. Apart from the above illustration, there are other illustrations of dual theories in physics. For instance, string theories exhibits T-duality and Ads/CFT duality is present between a field theory and a string theory (Polchinski 2017).

3.8 Duality in Other Branches of Science

The concept of duality is found in other branches of science too. For instance, in biochemistry, certain molecules are said to exhibit "functional duality" when they are the key initiators of not one but two unique processes (McCafferty et al. 1999). Similarly, "mechanistic duality" is formulated in chemistry to capture the dual ways — either homolytically or heterolytically — in which a chemical reaction can proceed (Banerjee 1997). In ecology, the "biotope space" (the physical space in which a species is found) and the "niche space" (an abstract space defined uniquely for a species) are supposed to exhibit a reciprocal relation that is termed as "Hutchinson's duality" (Colwell and Rangel 2009). In electrical engineering, electrical equations exhibit duality under certain transformation rules. In a equation, when voltage terms are replaced by current terms, resistance term with conductance, capacitance with inductance, etc., the resulting equation is equivalent to the original equation (Peelo 2014). In mechanical engineering, structural mechanics formulates similar kind of duality: the equations derived through the kinematic analysis of rigid body systems and the ones through the static analysis exhibit static-kinematic duality (Carpinteri 2014).

3.9 Duality and Symmetry

In the above sections, duality found in various disciplines have been discussed. This discussion highlights the multiple connotations of the concept of duality: right from the simplest meaning of there being a pair of entities to a much more complex definition involving some sort of equivalence amidst the pair. Leaving aside the cardinal duality, all the other types of duality share the similar structure of duality that can be represented schematically as $D \leftrightarrow D'$ indicating the duals — D and D' — connected through the relation of duality. Even though "$\leftrightarrow$" gets interpreted according to the context, is it possible to say something about the nature of this relationship?

It can be noticed that none of the instances of duality discussed above instantiate strict equality between their duals. Instead, in some instances (like the ones observed in classical logic, projective geometry and dual theories of physics), closer cognates of equality — equivalence or similarity — are found. Also, not surprisingly, the other extreme form of relation — the duals being contrary of one another — is also not observed in the spectrum of connotations discussed here. The relation between the contrary duals, like good-bad, hot-cold, dead-alive, etc., is often understood through the concept of negation. According to this interpretation, the pairs are immediate contraries and the negation operator transforms one to the other (Horn and Wansing 2017).[8] It is evident that none of the version of dualities considered here can be understood as a pair of immediate contraries related through the operation of negation. Among the instances discussed above, dualities found in classical logic, projective geometry, group theory and category theory, and dual theories of physics do involve a kind of inversion or reversal. However the inversion relation involved in these cases is quite unlike the strict form of negation. Therefore, the non-obvious instances of duality discussed in this paper do not seem to take either of the extreme connotations of "being equal" or "being opposite". Instead, duals are characterised by the weaker variants of these extremities: either the duals are similar to one another or are antithetical in a peculiar way. Apart from these two kinds, there are some duality relations which have both the characteristics. That is, neither of the characteristics — similarity or inversion — individually capture the essence of the duality relations found in logic, projective geometry and dual theories in physics. The duals in these cases, so to speak, occupies a liminal region between the extremes of equality and opposition such that they are not completely different from one another but at the same time are related through some sort of inversion. In this kind of duality, it appears that the transformation plays an important role since it brings about the dual characteristics of the relation: the duals that are similar to one another becomes inverse (in a specific sense) after the transformation.

This clarity about duality highlights the similarity between duality and symmetry since symmetry is also contextualised under specific transformations. This similarity suggests the possibility of interpreting duality within the larger framework of symmetry. However, an attempt to interpret duality along this lines is not possible if symmetry is defined, as

8. The qualification 'immediate' here is important because some contraries, like weak-strong and tall-short, can have intermediary values between the two extremes.

it is often done, as invariance that a thing exhibits under certain transformations. This interpretation of symmetry is not a good starting point because it portrays symmetry as a property of an entity unlike duality that involves a pair of entities. In contrast to this, I wish to take an alternative path to present the concept of symmetry through which the overlap of duality and symmetry becomes evident. Here, I want to focus on the historical argument presented by Hon and Goldstein (2008): in the late eighteenth century, a novel definition of symmetry was proposed by Adrien-Marie Legendre; prior to this, the notion of symmetry was understood as a property of an entity; in contrast to this, Legendre defined symmetry as a relation between two distinct entities (2008, 48). This historical shift of the concept symmetry, as I will show below, provides the required ground for considering the similarity between symmetry and duality.

In Greek thought, symmetry as a concept was used to indicate the proportionality and commensurability of two quantities (ibid., 2). It was only with Vitruvius, during the first century BCE, symmetry became a characteristic of the whole representing the correspondence among its parts (ibid., 101). In the seventeenth century, the correspondence among the parts of a whole and the equality of them were distinguished. This clarity about symmetry led to the development of the well known example of bilateral symmetry (ibid., 130). Throughout this time, the core meaning of symmetry, as a property of the whole and its parts, did not change. It was only due to certain developments in the late eighteenth and nineteenth century that apart from the established notion of invariance of the whole under certain transformation, symmetry also stood for equivalence among independent individuals. The development of this new meaning is historically contextualised in the difficulties encountered regarding the notion of equality in solid geometry. The accepted definition of equality for solid figures (i.e., three dimensional figures) was given by Euclid, who based the definition in turn on the notion of equality in planar geometry. According to this definition, two solid figures, like tetrahedrons, are equal and similar if they are "contained by similar planes equal in multitude and in magnitude" (ibid., 222). However, it was realised that this principle of equality is not sufficient because equality of planes that constitute two tetrahedrons do not guarantee their superposability. This problem arises because it is not only the equality of planes, but also the order of their arrangement to constitute tetrahedron plays an important role in deciding the solid-angle formed (ibid., 229). This situation was a concern for geometricians since it was not clear whether Euclid's principle is incorrect or is it incomplete and unsuitable for solid geometry. In the eighteenth century, Legendre attempted to resolve this problem by proposing a new notion of equality. For him, two tetrahedron are *equal* only if they are equal in magnitude and are also congruent (i.e., superposable). In contrast to these, the figures which are not superposable, Legendre argued, are still equal, but the principle at work is *equality by symmetry* (ibid., 236). By proposing this novel definition of equality in order to resolve the problem regarding solid figures, Legendre brought a change in the notion of symmetry.

Prior to this usage, symmetry referred to an intrinsic property of an entity and the configuration of its parts. With Legendre's principle, symmetry became a relation that relates two distinct entities.

This historical development regarding symmetry has lot of aspects similar to the notions of duality discussed in this chapter. The duality relation found between duals of projective geometry or category theory is similar to the symmetrical relationship between the two incongruent tetrahedrons formed by parts of equal magnitude. Like these duals, the two figures are not different but yet opposite to one another.[9] This overlap indicates the possibility of considering duality relation in general to be similar to symmetry relation. However, does this imply that duality can be understood completely through the notion of symmetry? Even though there are similarities, these two concepts are different. The similarity between duality and symmetry was grounded on the importance of transformations for both these concepts. Regarding symmetry, two entities are symmetrical if there is a transformation that relates these two through some invariance. In the context of duality, when the examples discussed above considered, the related transformations only show or demonstrate how the duals are related in a particular way. This implies that for duality, transformation is not necessary. There is also another important argument for not reducing duality to its transformation. The interpretation of duality through its transformation projects it as a relation that acquires two different characteristics before and after the transformation. This is an incorrect representation since the liminal duality relation does not switch its nature in this fashion. Instead, the relation possesses these characteristics simultaneously. This aspect of duals - to be simultaneously similar and inverse to one another - is an essential feature of duality and to provide its meaning is to explicate the nature of this feature.[10]

9. In fact, in contrast to a tetrahedron, the other figure's plane angles are supposed to be in "inverse order" (Hon and Goldstein 2008, 233).

10. Sundar Sarukkai and I have extended the analysis of duality discussed here in Bhatta and Sarukkai (2020), where duality is also analysed using the notion of completeness.

Chapter 4

Wave-Particle Duality and Light

The aim of the present chapter is twofold. The primary intention is to show how the notion of wave-particle duality has played a significant role in our quest to understand light. However, exhibiting this is not a straightforward task owing to the difficulties of pointing out what “wave-particle duality” means. Given the tumultuous history that radiation underwent through in the preceding century, it is not surprising that there is lot of confusion surrounding the concept “wave-particle duality”. In order to highlight this situation, in the first section of this chapter, I provide a survey of various contemporary scientific texts and exhibit the plural ways in which wave-particle duality has been defined. Subsequent to this, I provide few historical clarifications about the plurality associated with this concept and compare this plurality with other notions of plurality found in science. After this, I resume the main task to emphasise the role wave-particle duality concept has played in the twentieth century’s explorations of light. I will carry this out by presenting the three important interpretations of wave-particle duality: the physical interpretation of light by Einstein, the analysis of duality by Bohr within complementarity and the theoretical equivalence between the two views proposed by Dirac, Heisenberg and others.[1]

4.1 Plurality of Wave-Particle Duality

4.1.1 Introduction

Wave-particle duality has been one of the central concepts of the twentieth century physics.[2] This concept was so pivotal for the initial development of quantum mechanics

1. Since the focus of the thesis is duality of radiation and photons, an in depth analysis of these three interpretations — given their relevance for the history and the present theoretical and experimental analysis of the concerned phenomena — is necessary. Contrast to these views, the other notable interpretations — like that of Louis de Broglie and Bohm, and also Born’s interpretation — have not been considered in detail. Even though these latter views are historically important, these have played a significant role in the duality phenomena pertaining to matter (like that of electrons).

2. The discussions in this section appears in Bhatta (2020).

that the well-known historian of science Max Jammer suggests that the "birthday" of the theory should be determined based on the first proposal of the wave-particle duality (Jammer 1966, 46).[3] Being a widely discussed topic, at present there are multiple views subsumed under the title "wave-particle duality". For some, duality is a question that arose during the first three decades of the previous century and was positively resolved by quantum electrodynamics (QED). As a brief review on the presence of duality in the *Nobel Archive* mentions, QED — whose inventors were awarded the Nobel prize for "solving the duality problem" — goes "beyond the everyday dialectic of wave and particle duality to the synthesis of a quantum field" (Ekspong 1999, 48). According to this view, duality is nothing more than an important event in the historical development of quantum mechanics. Contrary to this, some consider duality as an intrinsic characteristic of the quantum domain. One of the prominent articulation of this stance has been the de Broglie-Bohm interpretation, according to which both matter and radiation physically have the dual characteristics. This view, which did not receive enough attention initially due to the Copenhagen "hegemony", has gathered support in the last few decades (Cushing 1994). In spite of this revival, the Copenhagen interpretation, based on the principle of complementarity, is still the dominant view about duality. This interpretation is still an active research topic where the principle gets verified in novel scenarios (see Wang et al. (2019)) and experiments on new entities are still conducted to observe the wave-particle complementarity (for example, see Kolesov et al. (2009)).

As the above overview highlights, this concept — being almost a century old — has accrued several meanings over time. Because of this, some historians have even concluded that "the wave-particle duality was and remains a rather vague concept that has neither been well defined nor used with sufficient consistency" (Kojevnikov 2002, 182). A scientific concept having multiple connotations is not unfamiliar. This characteristic is observed even in the case of fundamental concepts like mass and space. And, in a recent article, the same aspect of the general concept of duality was demonstrated (Bhatta and Sarukkai 2020). As these examples indicate, scientific concepts possessing multiple meanings is not unusual and also not problematic as the intended meaning becomes evident in the specific context of use. Even though this is largely true of wave-particle duality as well, the plurality associated with it has not received considerable attention. As pointed out above, wave-particle duality is depicted in some scientific texts as an obsolete historical artefact and in others, as a fundamental aspect of quantum entities. Not recognising this situation about the concept is a point of concern since duality still plays an important role in the pedagogy of quantum mechanics.

Given this, I want to highlight the plurality of wave-particle duality and discuss

3. According to Jammer, duality was implicitly contained in the new radiation law proposed by Planck ([1900a] 1967). Therefore, Jammer proposes the paper's publication date — October 19 (1900) — to be the day quantum theory came into existence (Jammer 1966, 46).

few of the important characteristics of this plurality. I want to begin this discussion by introducing, in section 4.1.2, the various historical proposals that are generally identified as wave-particle duality claims. With these views laid out, in section 4.1.3, I illustrate the plurality by presenting a broad survey of contemporary science textbooks and exhibiting how variedly duality is dealt in these texts. There are various reasons because of which the plurality of duality is not recognised. Among these, an important one has to do with the way the historical development of duality is narrated. In section 4.1.4, I evaluate the usual portrayal of duality as this specific concept going through different stages of development and show how it overlooks the plurality present in the history of quantum mechanics. In section 4.1.5, I engage positively with the nature of this plurality by comparing it with other pluralities found within science. Finally, in section 4.1.6, I conclude by emphasising the importance of this study for scientific research and pedagogy.

4.1.2 Multiple Historical Interpretations

Before illustrating the plurality of wave-particle duality in contemporary physics, I want to discuss few interpretations in the history of quantum mechanics that have been considered as proposals of wave-particle duality. This discussion about the prominent views — that of Einstein, de Broglie, Born, Bohr and Heisenberg — sets the primary ground for highlighting the current plural interpretations about duality since the contemporary plurality can be considered as the outcome of the multiple sources present in the history.

Most of the historical works on duality and quantum mechanics in general consider the conclusion present in the 1909 paper of Einstein as the first formulation of wave-particle duality (Klein 1964; Jammer 1966, 37; Hendry 1980; Pais 1986, 248). In this paper, Einstein attempts to understand the physical "constitution" of radiation with Planck's distribution law as the starting point. Through the analysis of momentum fluctuation equation, Einstein argues that radiation exhibits two different behaviours: at lower frequency, radiation behaves wave-like and at higher range, radiation acts as if it is constituted by "very small-sized complexes of energy" (Einstein [1909] 1989, 393). Given the influence this analysis had on the theoretical development in the later decades, Darrigol (1986, 199) has labelled it as the "most famous and universal puzzle of radiation theory". For Einstein, however, radiation showing these two "structural properties" was not paradoxical and he even comes up with a physical interpretation to accommodate these features, according to which radiation is nothing but energy singularities surrounded by force fields (Einstein [1909] 1989, 394).

The above analysis of Einstein, along with his other works, was the seed for other wave-particle duality proposals. One of these is that of Louis de Broglie. Unlike Einstein's inference that was limited to radiation alone, de Broglie made the bold move and raised duality to a fundamental principle. According to this, physical entities — both light and matter — possess the dual characteristics. de Broglie's initial research is historically rooted in the early twentieth century challenges concerning the physical structure of X-rays (Wheaton 1991). While working on these questions, de Broglie attempted to bring together two fundamental proposals of Einstein: the theory of relativity and the hypothesis of light-quanta (ibid., 286). Through this analysis, he proposed in 1923 that every material particle possesses an internal periodic phenomenon which is in sync with the "phase waves" surrounding the particle (Jammer 1966, 243; Darrigol 1986, 203). In the following year, de Broglie was able to work out a theory which exhibits the complete equivalence between matter and light (de Broglie [1925] 1963). In this matured theory, a particle is guided by the phase waves such that these waves determines the probability of the particle's position by exerting "quantum force" on it (Home 1997, 38-39). This theory, which did not receive much attention, was rejuvenated after Bohm's contribution in 1952 (Bohm 1952; Cushing 1996).

The other notable duality interpretation stemming from Einstein's proposal is that of Born's. This theory provides a new perspective about the wave aspects of particles. While analysing the diffraction pattern observed in the case of electron collision experiments, Born suggested that the Schrödinger's wave function should be interpreted as providing the probability density for the distribution of electrons on the screen (Born 1926b, 1926c, [1926a] 1983). In this formulation, the "probability waves" associated with the individual particles give rise to the wave-like pattern observed at the ensemble level. As Born recalls much later, the source for this interpretation was Einstein's physical rendition of light proposed in the 1909 paper (Jammer 1966, 41). The parallel between these two duality theses can be seen in the following description provided by Born (1926c): "the guiding field which is represented by a scalar function Ψ... spreads according to Schrödinger's differential equation. Energy and momentum, however, are transferred as if corpuscles were really flying around" (quoted in Falkenburg (2007, 269)). In this sense, the dual aspects of electrons are distinctly observed in the diffraction experiment: the particle feature is seen when individual electrons are detected and the wave aspect becomes evident at the ensemble level.

Apart from the series of duality proposals arising from Einstein's view, there are two other dominant interpretations of wave-particle duality: Bohr's and Heisenberg's. The view held by Bohr is contextualised in the principle of complementarity that he proposed in the 1927 Como lecture. Here, his main concern was about the "fundamental limitation" of the classical physical ideas for atomic phenomena. To highlight this, he shows how in the quantum theory, the "space-time co-ordination" and the "claim of causality" turn out to be "complementary but exclusive features of the description" (Bohr 1928, 580). Even though in this lecture he does not explicitly discuss about duality, his discussions with other physicists during that time clearly bring his views of wave-particle complementarity. For instance, in his response to Heisenberg's 1927 paper on uncertainty principle, Bohr uses the gamma-ray thought experiment to illustrate how wave and particle interpretations of light are mutually exclusive (Beller 1992; Camilleri 2006).

Another influential interpretation of duality is that of Heisenberg's, which was first published in his 1929 German paper. This view gets further articulated in his book, where he mentions "the problem of quantum theory centers on the fact that the particle picture and the wave picture are merely two different aspects of one and the same physical reality. Although this is a problem of purely physical nature it is satisfying to find a counterpart to this duality in the mathematical apparatus of the theory... one and the same set of mathematical equations can be interpreted at will in terms of either picture" (Heisenberg [1930] 1949, 177-78). As this quoted passage clarifies, for Heisenberg, duality is a situation where there are two equivalent mathematical formulations for describing the same quantum phenomenon (Camilleri 2006).[4] Similar view about duality was held by Dirac. This view is latently present in his 1927 paper, where he shows that both the physical interpretations of radiation — as a series of waves and as a stream of particles — provide the same expression for the hamiltonian of the system. By demonstrating this, he concludes that "the wave point of view is thus consistent with the light-quantum point of view" (Dirac 1927, 263; Bromberg 1977).

4.1.3 Current Plural Views about Duality

The above views of wave-particle duality are not obsolete. Most of the articulations of duality found in the current scientific literature, as I will show below, are endorsements of the above interpretations. However, the problem is that the present discussions hold on to one of the above views and overlook the others. Moreover, these discussions project different attitudes about the concept: some emphasise the need and relevance of this concept for the contemporary physics; others think otherwise. Thus, there is considerable ambivalence surrounding the concept of wave-particle duality at present. This situation becomes apparent when duality's depiction in the current scientific literature is observed. And it is this that I intend to carry out in this section. However, an attempt to provide a survey — even a non-exhaustive one — of a concept that has percolated deeply within the discipline is fraught with several hurdles. There are an abundant number of texts on this topic, not only the scientific ones, but historical and philosophical ones. And in this corpus, there are numerous views about duality, apart from the ones discussed in the previous section.[5] Given these difficulties, since my intention is to just indicate the prevalence of plural views, I will survey few of the prominent scientific pedagogic texts

4. Not that this proposal of theoretical equivalence went unscrutinised. Physicist Alfred Landé raised several theoretical concerns against this view (Landé 1959, 1962, 1971).

5. Even though the interest about duality might have subdued within physics, this is still an active research topic in the domain of history and philosophy of science. Works not only on reinterpretation of older views (see for instance Dewdney et al. (1992)) but also novel resolutions of duality are being proposed (see N. Maxwell (1988) and Suárez (2007)).

like textbooks and book chapters, and highlight how they differ in their interpretation about duality.

I want to begin the survey by noting that wave-particle duality is not ubiquitously mentioned or discussed in scientific texts. There are several texts that do not invoke duality. For example, there is no mention of duality in the well known graduate level textbooks on quantum mechanics like Landau and Lifshitz (1977) and Sakurai and Napolitano (2011). And there are others, for instance Schiff (1955, 15), which mention duality passingly without pausing to say anything meaningful about it. Apart from this set, there are ample well-known contemporary scientific textbooks that do discuss this concept. Majority of these books formulate duality either within Born's formalism or introduce Bohr's interpretation. Let me first review the books discussing Born's notion. Shankar, in his well known textbook on quantum mechanics, defines duality using Born's theoretical vocabulary: "each particle has associated with it a wave function $\Psi(x,t)$, such that $|\Psi(x,t)|^2$ give the probability of finding it at a point x at time t. This is called wave-particle duality" (1994, 113). Similarly, the book by Cohen-Tannoudji on quantum mechanics states "We can summarize [wave-particle duality] schematically as follows: (i) The particle and wave aspects of light are inseparable. Light behaves simultaneously like a wave and like a flux of particles, the wave enabling us to calculate the probability of the manifestation of a particle. (ii) Predictions about the behaviour of a photon can only be probabilistic..." (1991, 14). Even though Feynman does not invoke the phrase "wave-particle duality" in his lectures on quantum mechanics, he does mention how an electron acts "sometimes like a particle and sometimes like a wave" in the double-slit thought experiment and ends by providing the probabilistic interpretation of electrons' behaviour (2010, 1-10). Ballentine, while introducing the structure of his book, mentions how the derivation of the non-commutative aspects of quantum operators based on symmetry principles "replaces the heuristic but inconclusive arguments based upon analogy and wave–particle duality, which so frustrate the serious student..." (1998, xiii). Later, in a chapter while analysing the diffraction experiments of particles using interference of probability densities, he mentions "The interpretation suggested by this analysis is best described by the phrase wave–particle duality. It suggests that there is a wave associated with a particle, although the nature of the association is not entirely clear..." (ibid., 134).

The above texts do not discuss Bohr's notion of duality and this is not surprising given that most of these do not mention the complementarity principle either. Cohen-Tannoudji (1991, 50) does mention complementarity once passingly and Ballentine (1998, 5) refers to it negatively in the introduction. However, there are other set of texts that present duality formulated in the context of the principle. In most of these books, complementarity and duality are considered equivalent to each other. For instance, the book by Nouredine Zettili (2009, 26) has a sub-section titled "Wave–Particle Duality: Complementarity", in which the principle of complementarity is introduced. Apart these, as mentioned earlier, wave-particle complementarity is still widely used in the experimental domain. For instance, the demonstration of wave-particle complementarity of photons by Grangier et al. (1986) is a well-known recent instance. Also, the demonstration by Scully and Walther (1989) have influenced a series of "which-path" experiments that highlight the complex relation between duality and measurement. As these usages indicate, due to the close conceptual association between Bohr's proposal and the notion of measurement in quantum mechanics, duality gets articulated through the related concepts as well. To illustrate this, consider the introductory textbook on quantum mechanics by A. C. Phillips (2013, 15). Here, there is a sub-section titled "Measurement and wave-particle duality" in which the author does not define duality, but instead discusses "how the concepts of measurement and uncertainty can be used to provide a logical and consistent description of the wave-particle properties of quantum particles". Even though this discussion describes how the act of measurement decides what is observed in a double-slit experiment, there is no mention of the complementarity principle in the whole book.

In contrast to the above collection, there are other texts that explicitly dismiss the concept of duality solely because they find the complementarity principle either trivial or invalid. For instance, the well-known textbook by Griffiths (2005) mentions in a footnote: "The so-called wave-particle duality, which Niels Bohr elevated to the status of a cosmic principle (complementarity), makes electrons sound like unpredictable adolescents, who sometimes behave like adults, and sometimes, for no particular reason, like children. I prefer to avoid such language" (ibid., 420-21n1). Another relevant book to discuss here is by Dipankar Home (1997). Even though the book does mention Einstein's notion of wave-particle duality from the historical perspective (ibid., 372), the central discussion about the concept is carried out in the chapter "Wave particle duality of light and complementarity". In this chapter, Home discusses the problems pertaining to the complementarity principle and thus in turn about the duality concept based on it. According to Home, "The Bohrian interpretation of wave particle dualism stems from the consideration that apart from formal predictions of observed results, some intuitive understanding is also required in terms of classically visualisable pictures of particles and waves" and thus "if we remain confined within the formalism of quantum theory without demanding a visualisable understanding, the problem of wave particle duality ceases to be relevant" (ibid., 271-72). Home goes on to point out "the conceptual as well as empirical inadequacies of the Bohrian perception of wave particle complementarity" and argues how "a rational synthesis between wave and particle pictures" is possible (ibid., 299). Apart from these texts, a strong voice against complementarity and duality was raised by Lamb (1995), in a paper which argues for the redundancy of photons in modern physics. He states that duality "may be necessary for those who are unwilling or unable to acquire an understanding of the theory. However, this concept is even more pointlessly introduced in discussions of problems in the quantum theory or radiation. . . The 'Complementarity Principle' and the notion of wave-particle duality were introduced by N. Bohr in 1927. They reflect the fact that he mostly dealt with theoretical and philosophical concepts, and left the detailed work to post-doctoral assistants. It is very likely that Bohr never, by himself, made a significant quantum-mechanical calculation after the formulation of quantum mechanics in 1925-1926" (Lamb 1995, 84).

As de Broglie-Bohm theory was sidelined for a long time[6], it is only in recent times that discussions about this theory and related pedagogic materials are becoming available. Even within this short duration, however, there seems to be several views about duality found within this theory. For instance, consider the following two essays present in an edited volume about Bohemian mechanics. Dewdney and Horton (1996, 170) are of the opinion that "in the pilot-wave theory the long-standing interpretative puzzles, for example the measurement problem and wave-particle duality, are resolved simply by completing the quantum formalism through the postulation of individuals (the particles) which maintain their identity through the continuity of their space-time trajectories". In contrast to this interpretation, Fine (1996, 239) thinks that unlike Bohrian position which "merely flirts with dualism but avoids commitment, Bohmian mechanics embraces it. Bohmian mechanics requires both wave (= Ψ function) and particle (= position coordinates) in order to specify the state of a system". After this initial suggestion, Fine further goes on to provide an open ended analysis by suggesting that the theory can either be interpreted "dualistically as involving the same old things, a one-way dualism of waves guiding particles" or "monistically as involving a new kind of unitary world-stuff" (1996, 240). Compared to these discussions of duality, a recent textbook on de Broglie-Bohm theory by Bricmont (2016) does not discuss wave-particle duality.

Apart from the above ones, there has been another interpretation of duality contextualised in field theories, like quantum electrodynamics (QED). One of the open questions about these theories pertains to their ontology: are both field and quanta required or are fields physically fundamental? Given that this theoretical enquiry has overlap with the historical context of wave-particle duality, it is not surprising that the concept of duality was used in this situation as well. For instance, a review paper on the history of photons mentions: "The duality of light, coupled with the corpuscular photon model, has been given many conflicting interpretations and has promoted almost universal confusion among nonexperts" (Kidd et al. 1989, 27). In this sense, some physicists consider the "field-particle duality" is continuation of the older debate about the wave-particle duality (Hobson 2013). Even though the question about the need of both photons and fields is categorically different from the other notions of duality discussed above, the proponents of field theories largely consider the subsequent development of QED resolved wave-particle duality. As Mario Bunge states, "the wave-particle duality ... stimulated the creation of another theory, QED, that did away with that duality ... The optical duality is then a

6. see Perillán (2018) for more on this

relic of the 1905-1927 interregnum" (Bunge 1968, 271). In another review article on the concept of photons, Scully and Sargent (1972, 47) mention "there is no need to switch from quantum to classical descriptions or to introduce a mysterious wave-particle dualism in order to explain interference and diffraction". Most of these arguments are based on the success of Dirac's theory to show the equivalence between both the wave and particle formulations of the system. It is important to note this because quite opposite to these opinions, Dirac "always kept a relaxed attitude to the wave-particle problem" and in a interview conducted in 1982, he reiterated the view discussed in the previous section: "One can treat light as composed of electromagnetic waves, each wave to be treated like an oscillator; alternatively, one can treat light as composed of photons, the photons being bosons and each photon state corresponding to one of the oscillators of the electromagnetic field. One then has the reconciliation of the wave and corpuscular theories of light. They are just two mathematical descriptions of the same physical reality" (quoted in (Kragh 1990, 338n40)).

4.1.4 Drawbacks of Unified Narrations

The above discussion illustrates the various ways in which duality is defined at present. Each of these definitions is situated in a unique theoretical context that has a different criterion of what counts as a valid experimental observation of duality. In spite of the obvious distinctions among these views, the plurality associated with duality has been overlooked. Among the many reasons for this, a prominent one is the presumption that this plurality is nothing but the accretion of multiple interpretations, offered at different stages, as the concept underwent gradual maturation. In this section, I want to show that this kind of unified narration provided by several reviews gets certain aspects about the history of the concept wrong.

Among the several historical works on wave-particle duality[7], there are some that depict a linear narration of wave-particle duality undergoing a continuous development, from being a tentative proposal to a well-confirmed hypothesis. Consider, for instance, the review of duality provided by Milonni (1984). This essay starts with Einstein's 1909 paper, in which he is supposed to have "provided the first clear indication of the wave-particle duality of light", subsequently discusses the 1926 paper of Born, Heisenberg and Jordan, and ends with the analysis of Dirac's 1927 contribution by noting that this theory "incorporates naturally the wave-particle duality of light" (ibid., 42). This way

7. The history of wave-particle duality has been covered in various works. Among these, Jammer (1966) and Pais (1986) have not paid specific attention to the nuanced differences among the views associated with duality as they focus on the larger development of quantum mechanics. Other historical works — like Klein (1964), Hendry (1980), Wheaton (1991), Dongen (2007), Duncan and Janssen (2008) and Fick and Kant (2009) — focus on a particular interpretation of duality situated in a specific context. Except for few — like Kojevnikov (2002) and Falkenburg (2007) — none of the works have paid attention to the historical circulation of the concept and the variety of ways in which duality was invoked.

of connecting the important junctures of theoretical development portrays the typical progression of a concept through stages like proposal, reinterpretation and resolution. Other surveys construct similar picture, but with different narrations. To mention one, consider the review by Combourieu and Rauch (1992). This work covers, along with the relevant historical events, the modern experimental efforts that examine duality. In the historical summary, the authors start with Einstein's papers and discuss de Broglie's "dualist principle". In a section titled "1924-1927: The Multiplicity of Dualist Interpretations . . . ", the authors summarise differing "views on the way in which waves and particles are articulated in the electromagnetic field" and subsequently moves to Bohr's formulation of complementarity principle, which they label as "the official version of dualism" (ibid., 1413-17). This authoritative characterisation of Bohr's formulation is re-emphasised at several places in the paper: they mention how "the complementarity principle found its mathematical justification . . . with the Heisenberg indeterminacy relations" and conclude the discussion by stating "this has been the majority attitude of physicists since that time" (ibid., 1418). Thus, the authors build a narration where duality concept goes through various formulations before acquiring the matured form.

The first thing to note about the above reviews is that none of them define duality explicitly and they take for granted what this concept means. Moreover, there are several problems with these narrations. First of all, this kind of portrayal implies that there was this definite concept called wave-particle duality which was present right from the beginning for the physicists to refer and deliberate upon it. Also, this narration paints a linear development of the concept, as if there was a single problem right from the beginning that gets resolved eventually. However, when the history of quantum mechanics is closely scrutinised, these presumptions get challenged.

The above discussed reviews, by providing the narration of how the early physicists analysed this concept and how it underwent subsequent changes, presume that there was this specific concept labelled as "wave-particle duality" available during this time. However, it appears that none of the initial prominent proponents of duality — Einstein, de Broglie, Bohr and Dirac — use phrases like "wave-particle duality", "duality" or "dualism" to refer to their respective initial proposals. In Einstein's 1909 German paper, there is no mention of cognates of "duality". Instead, the English translation of the central idea that Einstein proposes is: "a theory of light that can be understood as a kind of *fusion* of the wave and emission theories of light" ([1909] 1989, 379, emphasis added). In fact, according to Alexei Kojevnikov, "Einstein did not use the word 'duality' either before or after 1925, nor did he make any clear assertion of the principle of the wave-particle duality" (2002, 182). In the case of de Broglie, his 1924 PhD thesis also does not contain any direct invocation of "duality". The closest reference in this work to duality is him discussing, under the section *The motion of an atom of light*, how the coincidence of "light wave" and "phase wave" for these atoms "evokes the *double aspect* of particle and wave" (de Broglie [1924] 2004, 41, emphasis added).[8] Similarly, Bohr (1928) and Dirac (1927) do not mention any of the cognates related to wave-particle duality. The absence of the phrase "wave-particle duality" or its cognates in the initial set of texts suggests that none of its proponents initially associated or identified their views as *duality* proposals to begin with. This implies that the consideration of these being "duality" interpretations must have been later ascriptions. Since "duality" was not used by the initial proponents during the formulation of their proposals, it would be whiggish to consider these as expressions of *wave-particle duality* at a later point of time (Brush 1995, 219).[9] Thus, without being careful about historiographic guidelines, the initial views were labelled as "duality" interpretations. Nonetheless, the whiggish accusation has limitation since some of the initial proponents — like Bohr, de Broglie, Dirac — subsequently adopted the label of "wave-particle duality" to talk about their views.

Wave-particle duality and its cognates, like "dual" and "dualism", started emerging in the English scientific literature only towards the end of 1920s.[10] With respect to the presence of duality in German literature, Kojevnikov observes that "dualismus" and "dualistae" — the German language concepts that are cognates of duality — were used in theology and philosophy since eighteenth century. He adds that "it is not easy to establish who first brought the term into physics. Neither Einstein nor de Broglie, to my knowledge, had used it, but by 1927, *Dualismus* has already been present in a number of German-language physical papers and typically attributed to Einstein's and de Broglie's views" (Kojevnikov 2002, 206, emphasis in the original). Here, it can be asked why "duality" came into usage and became popular only by 1927 in spite of the availability of Einstein's initial proposal in 1909? A response to this can be attempted by using some of the information that Kojevnikov provides in his analysis. As he notes, even though Einstein had suggested the physical interpretation of radiation having both the wave and particle aspects in 1909, it was not immediately recognised as "duality" proposal as this fluctuation formula could be interpreted in variety of ways, "duality being only one of them" (ibid., 183). And, since Einstein accepted duality "only in a negative sense", his attitude might not have been conducive for the rise of duality (ibid., 217). Moreover, it appears that the 1909 proposal of Einstein might not be the source for the emergence of duality. Kojevnikov

8. This statement is from the English translation of the original French text (de Broglie [1925] 1963). The phrase that is used in the original French text is also "double aspect".

9. Whiggism is the tendency to read the past with reference to ideas that became available only later. Contrast to the whiggish interpretation, contextual narratives should be preferred. Contextualism is the way of studying "the ideas and theories of a period in terms of the scientific knowledge and general culture of that period, without regard to what came afterwards" (Brush 1995, 219).

10. Here, it should be noted that there are numerous challenges while tracing the occurrence of wave-particle duality during the initial quarter of twentieth century. Other than the task of exploring the texts in different languages, investigating when the usage of duality actually began is not straightforward as there were multiple contexts apart from the wave-particle conflict where *duality* was the defining concept. For instance, "dual" characteristics were associated to radiation by J.J. Thomson for completely different reasons (Thomson [1928] 2016; McCormmach 1967). Along with that, the "duality" associated with x-rays and γ-rays has an independent origin and evolution (see Wheaton (1991)).

points that during the early 1920s, the physicists became aware of Einstein's idea of duality through his 1925 paper, where he proposes the Bose-Einstein statistics (Kojevnikov 2002, 217-18). Thus, only by accepting Einstein's proposal "in a positive sense, as one of the most basic principles of the new theory", the physicists of the "new quantum mechanics" like Schrödinger, Born, Jordan made "Einstein's reputation as a dualist became solidly established" (ibid., 209).

Post 1927, the notion of duality was not confined to Einstein's and de Broglie's views alone. Within a decade, duality is found in several papers having the range of connotations that this concept currently possess. For instance, in 1929 paper, Arthur Compton, mentions "the fundamental things in nature, matter and radiation, present to us a dual aspect. In certain ways they act like particles, in others like waves. The experiments tell us that we must seize both horns of the dilemma" (Compton 1929, 87). The phrase "wave-particle dualism" is found in a 1929 paper where the phrase refers to Compton's effect and the Davisson and Germer experiments (Ellett et al. 1929, 493). In a 1936 paper, the author notes how after de Broglie's proposal and discovery of Compton's effect, "much has been written and discussed on the wave-particle duality... by now it has been thoroughly worked into the structure of theoretical physics and has become a commonplace in our everyday discussions of atomic processes" (Hill 1936, 225). To cite another instance, a 1938 paper, which reviews a scientific exhibition at Paris, notes how a certain exhibit highlighted "the wave particle duality of electrons and of radiation based on the immortal experimental findings of Gray, Compton, Davisson and Germer, and G. P. Thomson together with its enunciation in Heisenberg's principle of indetermination..." (DuMond 1938, 293).

The above inferences should be considered as preliminary findings and further extensive historical survey needs to be carried out. Nevertheless, the brief review of the papers from the initial phase of development illuminates an important point: the plurality associated with wave-particle duality is not a recent phenomenon. Within a short span of time since its emergence, wave-particle duality had become a concept that is applicable to diverse theoretical and experimental interpretations. The early onset of plurality counters the narrations found in the reviews discussed above. The tendency to depict duality as a specific concept which reaches its final state has been noted by several historians. For instance, Mara Beller mentions "the story of the development of quantum mechanics builds dramatically around the wave-particle dilemma. We follow the piling up of contradictions between the corpuscular and wave aspects of matter and radiation, one after another, until the resolution of the ultimately unbearable conceptual tension by Bohr's principle of 'wave-particle complementarity'" (Beller 2001, 223). Against this type of unified narration, she argues that "when we read papers dealing with the wave-particle issue before the rise of the Copenhagen philosophy, we hardly find feelings of desperation or distress. A patient suspense of final judgement seems to be a more fitting characterization of the attitude of physicists". She further adds that not all scientists of that time felt that the situation demanded the need of two models. Some scientists thought that these two sets of observations were reconcilable; others felt this is just a temporary hurdle till proper theoretical resolution arrives (Beller 2001, 224-25).

4.1.5 Nature of Plurality

In the previous section, one way through which the plurality of duality gets misinterpreted was considered and critiqued. Apart from that, there are other factors because of which the plurality has not been sufficiently recognised. This is also because the nature of the plurality found in the context of wave-particle duality has not been positively dealt with. Given this, in this section, I want to first conceptually analyse the different notions of duality mentioned in the section 4.1.2 and establish how their distinctness ground the plurality. After that, I will characterise this plurality by considering different types of pluralities usually observed in science.

Distinction between Views

In the section 4.1.2, I presented some of the prominent views that are identified as "wave-particle duality" proposals. Historically, these views did not develop in isolation and as several historical works have shown, some of these proposals were formulated as a response to others. Having acknowledged that these are not independent views, I want to emphasise on their distinctness here. To begin with, the contexts of enquiry in the discussed proposals — for which each of these views were supposed to be the solution — do not overlap with one another. This dissimilarity can be seen by comparing the dominant duality views: Einstein's, Bohr's and Heisenberg's. In his 1909 paper, and even later on, Einstein struggled with the physical structure of radiation. The way he tried to answer this, at least in the 1909 paper, is by providing a novel physical interpretation of radiation. The problem Heisenberg was concerned about was the presence of two completely different yet equivalent theoretical formulations. In contrast to these, Bohr's preoccupation was regarding the larger conceptual vocabulary used to describe atomic phenomena. Thus, there was no common problem and each of the proponents were engaged in unique enquiries.

The other crucial difference among the three views is the meaning of duality. In these cases, even though the solution offered was called "wave-particle duality", the meaning of the concept is different in each context. When the three proposals are compared, the meaning of the terms "wave", "particle" and "duality" are found to be distinct. In Einstein's 1909 proposal, "wave" and "particle" are two different behaviours exhibited by radiation at different frequency ranges. However, in the context of the physical interpretation that he provides towards the end of the paper, "wave" and "particle" are physical features of radiation. If "duality" needs to be understood within this formulation, then it refers to the presence of two constituents of radiation. Bohr considered "wave" and "particle" as classical pictures or models that needs to be used for making sense of experimental observations. And "duality", in his framework of complementarity, acquires the meaning of "mutually exclusive, but jointly complete" (Held 1994, 871). In the case of Heisenberg, "wave" and "particle" refer to theoretical models, each of which belonging to completely different theories. Moreover, given the theoretical equivalence between them, he did not consider that both these pictures are essential for understanding quantum mechanics; either of the approaches are sufficient (Camilleri 2006, 310). As can be seen, in each of these views, duality is a unique concept that has nothing in common with the other definitions. Nevertheless, the above differences between these proposals should not come as a surprise given that each of the physicists belonged to different worldviews. Einstein was greatly influenced from the rivalry between mechanical and electromagnetic worldviews prevalent during his time (McCormmach 1970b, 1970a). Bohr, Heisenberg and others belong to a complete different worldview which eschewed mechanical concepts and embraced mathematical, abstract thinking (de Regt 2017; Camilleri 2007a).

Characteristics of Plurality

After having established the distinctness of the multiple views of wave-particle duality, I want to analyse the characteristics of the plurality which they constitute. This is of interest because not all instances of plurality found in science are of the same kind.[11] In this section, I will briefly introduce few kinds of pluralities through relevant examples and subsequently highlight some aspects of the plurality concerning wave-particle duality.

In science, the common instances of plurality are that of general and fundamental scientific concepts. The basic concepts like scientific method, theory, observation, etc., have unique interpretations in each of the sub-discipline of science. Apart from these general notions, fundamental theoretical concepts of every sub-discipline — like, for instance, physical notions like mass (Jammer 1997), force (Jammer 2012) and space (Jammer 2013) — also exhibit plurality. Even though recognising plurality is straightforward, in some cases establishing what makes the seemingly divergent notions "plural views" of the same concept can be an arduous task. Consider, for example, the plurality of mass. The notion of mass found in theory of relativity corresponds to Newtonian mass when velocity is not close to that of light. On the other hand, electromagnetic mass is categorically different and has nothing in common with these two notions. Nevertheless, there are cases of plurality, where the relation among the plural views is evident. This can be illustrated through the plurality usually associated with scientific objects. Consider the case of positron for instance. As Norwood Hanson mentions, "the discovery of the positive electron was a discovery of three different particles… (1) The Anderson Particle, (2) The

11. For a defence and a theory of plurality of concepts, see Weiskopf (2008).

Dirac Particle and (3) The Blackett-Occhialini Particle". Hanson argues that it is better "to put the matter this way, rather than to remark three different discoveries of the same particle, because the conceptual backgrounds within which the work of Dirac, Anderson, and Blackett took place were so disparate as to leave it unclear until almost 1934 whether their findings had anything in common" (Norwood Russell Hanson 1961, 194-95). Similar plurality has been attributed to electron (see Falconer (1987) and Bain (2004)) and photons (see Hentschel (2018, 39)). As all these case-studies highlight, holding onto the presence of a single unobservable entity across theory change and different experimental scenarios invariably give rise to plural views of that entity. However, what holds all the views together is the presumption that all these still refer to the same entity.

Contrary to the above one, another kind of plurality can be illustrated through the multiple definitions associated with the notion of gene. Right from its introduction, biologists have disagreed with one another about the meaning and function of gene. For some, it referred to an entity; and for others, gene served merely as a heuristic purpose in making sense of the observations. Given this, Raphael Falk (1986) identifies several notions of gene found in its historical development: instrumental gene, material gene, holistic gene and the DNA gene. Subsequently, Falk goes on to argue the difference between the plurality found in the case of genes and the one that is observed in the context of other entities like electron. He observes that unlike electron, which "changes its meaning but not its reference, the term 'gene' has changed both its meaning and its reference" (ibid., 170). Thus, this is an ideal example of non-simple instances of plurality where the commonality among the different views is not directly given and needs to be worked out.

The above discussion sets the ground for identifying few important characteristics of the plurality observed in the case of wave-particle duality. On one end, since wave-particle duality is not an essential theoretical concept of quantum mechanics — as some argue for doing away with this concept — its plurality is not as rich and complex as that of other fundamental concepts like mass and space. At the same time, the plurality of duality is also not straightforward as the one observed in the case of positrons. This is because the multiple views of duality are distinct and have no obvious common aspect. This plurality seems to be similar to the one observed in the case of genes: the multiple views of duality are not only distinct in their meaning, but also differ in what they pertain to. Given this, as in the case of genes and other interesting instances of plurality, the plurality of duality needs to be further analysed to understand the relation among the plural views.

4.1.6 Conclusion

In this section, I have highlighted the plurality of wave-particle duality and have brought forward few of its characteristics. The claim of plurality, when taken seriously, can not only provide better understanding of the concept, but also meaningfully guide the scientific research in this domain.[12] There have been few recent works that have successful shown how by acknowledging the multiple interpretations of duality results in meaningful clarifications about the concept. The works by Beller (2001), Camilleri (2006) and Camilleri (2007b) is worth mentioning here since they have argued against the common belief that Heisenberg and Bohr held similar views about duality. According to them, even though Heisenberg hesitantly agreed with Bohr during 1926-27, theoretical proposals of Jordan, Klein and Wigner in 1927-28 eventually changed his views to conclude that "wave" and "particle" are equivalent formulations. This kind of meticulous historical and philosophical effort is necessary given the complexity of the concept at hand. Overlooking these differences can lead to problematic conflations like "Einstein may be considered as the godfather of complementarity" (Pais 1986, 248). Another important contribution to note here is by Kojevnikov (2002). Apart from the historical clarifications which are already mentioned in the above sections, he also provides an interesting classification of the different views of wave-particle duality. According to him, these views belong to either of the two categories: one set of views — which he calls "dualism" — interprets "wave" and "particle" statements at the ontological level; the other position — which he recognises as "duality" — subscribes to "opportunistic freedom" to choose between different "languages" (ibid., 208).

In spite of these efforts to disentangle the complex narrations, duality is still being largely discussed without due care. An apt instance of misuse that still prevails is regarding the wave-particle complementarity, which still enjoys considerable support within the physics community. The current discussion on this view does not take into account the historical revisions made to this concept. During 1927, Bohr indeed interpreted the wave-particle problem using the complementarity principle. But post 1935, as Held (1994) has pointed out, Bohr no more considered wave-particle issue to be a valid complementarity situation. The main hurdle for Bohr was to articulate the mutual exclusivity of wave and particle observations. Also, it is not clear in what sense wave and particle pictures complement each other (Bokulich 2017). Few physicists too have experimentally demonstrated the weakness of complementarity. Ghose and Home (1992), for instance, show how both "wave" and "particle" features can be observed within the same experimental setup, much against the common belief about their complementarity. However, these critiques are yet

12. One of the examiners here pointed the following: "I am not convinced that rigorous adherence to proper definitions and unambiguous notions . . . has to be an a priori condition for progress in theoretical physics". In the thesis, neither have I discussed about scientific progress nor have mentioned that proper definitions and unambiguous notions are a prior conditions for the same. In fact, I completely concur with the examiner that such criterion cannot be held in any research endeavour (not only scientific) where addressing the query demands coming up with novel yet nascent and tentative concepts/hypotheses. Having said that, I do think, at every stage, a reflective introspection needs to be carried out to examine the current status and this kind of a posteriori analysis is crucial for the progress and health of any discipline. I think, the thesis incorporates both the above aspects (i) by highlighting how early physicists grappled with the difficult questions and provided creative conjectures and (ii) by suggesting how one thread of development (i.e., the duality concept in the context of photons' interference experiments) needs to be re-examined.

to make a meaningful impact on the current scientific research in this area.

To conclude, it cannot be denied that the present situation about duality is largely due to the inadequate way in which this concept is dealt in the pedagogy of quantum mechanics. There have already been many reports about this state of affairs (Cheong and Song 2014; Greca and Freire 2014). The survey of duality within the scientific literature and the preliminary characterisation of the plurality carried out above positively contribute towards addressing some of the concerns. As others have already opined (see Sarukkai (2014)), the current situation can also be alleviated by actively incorporating historical and philosophical perspectives in science pedagogy.

4.2 Duality and Physical Interpretations of Light

After surveying the plural views about duality, from this section, I will explore the prominent interpretations of duality and show how each of these interpretations brought significant insight about radiation. The first strand that I want to explore is the relation between wave-particle duality concept and the physical interpretations of light. This series of developments started with the very articulation of blackbody radiation law in 1900. Therefore, the first wave-particle duality proposal by Einstein in 1909 can be considered as the culmination of a decade long theoretical developments. In this section, all these key stages are discussed with the intention of elaborating the nature of duality in this larger context.

4.2.1 Planck's Law and Initial Analyses

The right place to start the story about wave-particle duality of light is with the discovery of the novel frequency distribution law by Max Planck. This law not only set the ground for the development of wave-particle duality, but also contained the seed for quantum mechanics. When this distribution law was initially proposed to suit the experimental results, this law could not be immediately given a theoretical derivation. Though Planck attempted to provided one, there were so many aspects of this derivation that either could not be understood within the framework used or lacked a sound physical interpretation. This difficulty in making sense of the law at that time was the early symptom of what would later be recognized as the paradoxical nature of radiation. Because of this, many historians have pointed that Planck's law contains "the essence of the wave-particle duality" (Jammer 1966, 46; Hendry 1980, 62). Apart from this historical interest, there is another important reason for considering Planck and other physicists' analyses of the new law here. The analysis of Planck's law — by Planck himself and many others — provides the first case study for how the context of wave-particle duality, even though being in the latent form during these analyses, became the principle guiding question that shaped the enquiry about radiation. In most of these analyses, the point of concern was the nature of discrete energy quantities, the important conceptual constituent of the wave-particle duality. So, one of the main challenge for the physicists during this period was to understand this novel proposal. In this section, I will highlight how Planck's and few other chose not to provide physical significance to this concept. For Planck, the emphasis was largely on the derivation of the theoretical relation and its clarification. This *theoretical mode of analysis* becomes evident at various places: in his attempts to generate "completely arbitrary relations" and evaluate their simplicity; in his interest to provide a deductive derivation of the distribution law. Apart from these specific qualifications he himself uses, I will also consider the way he analyses radiation-matter interaction and the various accompanying aspects he presumes in order to arrive at the distribution law. Specifically, I will discuss two stipulations that Planck adopts: the assumption of energy distributed over resonators being composed of finite elements and the method of calculating the probability of a state. In each of these cases, I will highlight how the motivation for adopting these stipulations are theoretical in nature and none of them correspond to the physical structure of radiation and matter. In other words, these specific aspects, which are crucial for Planck to arrive at the distribution law, do not have any physical significance; they merely have theoretical motivations.

Planck's Initial Proposal

Regarding the study of black-body radiation, by 1890s, the energy distribution law that accounted for all experimental results till then was given by Wien:

$$\rho(\nu, T) = \alpha\nu^3 \exp\left(\frac{-\beta\nu}{T}\right) \tag{4.1}$$

where, $\rho(\nu, T)$ is the energy distribution function of thermal radiation which is at an absolute temperature T and its frequency is ν; α and β are constants (Klein 1961, 461). Planck, who was attempting to reduce thermodynamics to Maxwell's theory, felt that the distribution law (4.1) could be adequately grounded in the theory of electromagnetism (ibid., 461). By late 1899, Planck had shown in series of papers that the same can be achieved and concluded that "the definition given for the entropy of radiation, and also the Wien distribution law for the energy which goes with it, is a necessary consequence of applying the principle of entropy increase to the electromagnetic theory of radiation" (ibid., 463). However, by late 1900, few important experiments showed that Wien distribution law does not capture the behaviour of radiation in the higher wavelength range. Following these results, Planck published a paper in which he suggests a new distribution law that fares better than Wien's law in corroborating with the the recent experimental results. By considering the interaction between radiation and matter through the model of n linear resonators in a stationary radiation field, Planck mentions his attempt to "construct completely arbitrary expressions" for describing the infinitesimal increase of the entropy of these oscillators (Planck [1900a] 1967, 80). Among these various expressions, he recommends a new distribution law based on two criteria: the simplicity of this expression compared to other expressions and its ability to match with experimental observations (ibid., 81). The suggested distribution law that accounts for the new experimental results is:

$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} \tag{4.2}$$

where k and h are universal natural constants (Klein 1961, 470). This new distribution equation corroborated neatly with the experiments results. In spite of this success, the "true physical meaning" (as Planck himself puts it) of this law was not clear (ibid., 468). In order to provide an understanding of this distribution law, Planck attempts to reinterpret this law within Boltzmann's statistical analysis. During this time, other physicists were also attempting to interpret the meaning of the law. Rayleigh shows that if Boltzmann's approach is adopted strictly, the distribution law would look like the following:

$$\rho(\nu, T) = \left(\frac{8\pi\nu^2}{c^3}\right)(kT) \tag{4.3}$$

It is interesting to observe that Planck's distribution law takes the form suggested by Rayleigh at lower frequencies of the radiation. When Planck's distribution law is compared with Wien's distribution law and Rayleigh's law, it is easier to see how (4.2) takes either the form of (4.1) or that of (4.3) depending on the frequency of radiation. This characteristic feature of (4.2) shows the duality inherent in Planck's law: the distribution law (4.3) is arrived through the wave-interpretation of radiation and Wien's law (4.1) can be made sense only with the particle interpretation of radiation. However, this inherent conflict present in the law was not evident during the point of conception and it was only much later, after several repetitive examinations, the duality was confronted. It is only in a retrospective analysis, therefore, that the label suggested by Hendry for this early phase as "intimations of duality" makes sense (Hendry 1980, 62).

Planck's Analysis of the Law

Towards the end of 1900, Planck published a subsequent paper in which he further elaborated on the distribution law he suggested two months back. The paper was a clarificatory one where Planck attempted to "derive deductively" the new distribution law from the basics of the electromagnetic theory of radiation (Planck [1900b] 1967, 83). He mentions that the right way to analyse the energy distributed over many resonators, within the framework of electromagnetic theory, demands the adoption of Boltzmann statistical approach. Planck models a system consisting of radiation and matter in the following manner: there are large number of resonators which are relatively independent from each other, but interacting with the radiation; among these resonators, N vibrating resonators are oscillating at frequency ν, N' are at ν' and so on. The total energy E_t is distributed as vibrational energy E_0 across all the resonators and the remaining energy is present within the radiation itself (Planck [1900b] 1967, 84). In this context, the central question (for which his distribution law is the answer) is to find the energy distribution across the resonators and radiation when the system is in the equilibrium state. Planck derives the required law by adopting Boltzmann's statistical approach, according to which the temperature of a state directly depends on the probability of the system being in that state. Boltzmann prescribes a specific way of calculating the probability of various states the system can be in. Here, finding the theoretical expression for the temperature of the equilibrium state becomes straightforward when it is considered that the equilibrium state has the highest probability among the various states. Planck, in this derivation, could not entirely adopt Boltzmann's approach since the context of these two enquiries were different: Boltzmann was analysing the thermodynamic state of an ideal gas; in contrast to this, Planck was interested in a system that involves the interaction between radiation and matter. Because of this, Planck had to suitably change and reinterpret Boltzmann's approach at two places in order to arrive at his desired law. It is important to consider these differences in bit more detail to bring out Planck's style of analysing these packets of energy.

One of the controversial presumptions that Planck proposes in his derivation is the presence of "energy elements" (ibid., 84). The presumption of these finite elements is stipulated by Planck while deriving the distribution law using the Boltzmann's approach. Since the statistical method specifies that the entropy of the system in a specific state depends on the probability of the system being in that state, Planck articulates this probability based on the following analysis. As mentioned earlier, the system is said to be made up of various resonators operating at different frequencies: N resonators at ν, N' at ν' and so on. Given a particular range of frequencies the system is operating at, E_0 can be considered to be distributed among various groups of resonators, each oscillating at a specific frequency. The total vibrational energy present among the resonators can be expressed as the following summation: energy E distributed across N resonators, E' associated with N' and so on. This various ways of distributing the resonators across these groups decides the probability of the system being in a particular state. This formulation still does not provide the complete description of the state of a group of resonators as the distribution of the available energy across the resonators of the group is yet unclear. This is because the energy E can be distributed in various ways across the N resonators oscillating at frequency ν. And, this task of finding the possible ways of distributing the energy is not straightforward if energy is considered as a "continuously divisible quantity", as this would result in infinite possible distributions. In order to circumvent this problem, Planck presumes that E is composed of "a very definite number of equal parts", where each finite element of energy is equal to $h\nu$ (Planck [1900b] 1967, 84). With this presumption, there are only finite possible ways in which the total number of "energy elements" P are distributed across a group of resonators.[13] It should be noted here that the main motivation to stipulate this presumption arises from theoretical concerns — to avoid dealing with infinite possibilities. Apart from this, there is no attempt by Planck in these papers to analyse the physical structure of radiation or the nature of physical interaction between radiation and matter (as could be seen in later enquiries regarding photoelectric effect and compton phenomenon) and provide physical significance for the notion of energy elements.

Planck's indifference towards the need of physical interpretation can also be highlighted by showing how he overlooks another important aspect of Boltzmann's stipulation while adopting it for his derivation. According to Boltzmann, the thermodynamic state of a volume of gas can be modelled as a system comprised of N_m molecules and the energy of each of these molecules is a discrete value such that these energy values form an arithmetic progression $0, \epsilon, 2\epsilon, \ldots M\epsilon$. Here, the discrete energy value ϵ is just an "artifice" since Boltzmann believes that energy associated with molecules cannot be physically discrete. This consideration is merely an intermediate step and at a specific stage of analysis, this presumption of discrete energy is removed such that molecules can possess any energy value in a continuum (Klein 1961, 473). Moreover, Boltzmann is clear from the beginning about the model of discrete energy values he uses in his model: "this fiction does not, to be sure, correspond to any realizable mechanical problem, but it is indeed a problem which is much easier to handle mathematically…" (ibid., 472).

The second important aspect of Planck's derivation pertains to the notion of probability that he stipulates. As discussed, the probability of finding the system in a specific state helps in knowing the temperature and entropy of the state. Given this importance of probability in Boltzmann's approach, Planck had to suitably define it for his enquiry. With the presupposition of energy elements, Planck finds a definite way to articulate the possible ways — called *complexions* — in which a specific amount of energy (E) might be distributed over a group of resonators (N) (Planck [1900b] 1967, 84). Each complexion then captures a different distribution of energy across resonators of a particular group. With complexions for each group thus calculated, the product of these complexions provides the total number of all possible complexions R over all resonators (ibid., 85). Here, Planck suggests that there is one distribution such that the total number of complexions R_0 for this is greater than that for any other possible distribution. It is this distribution that should be preferred when the system of resonators is in equilibrium with the stationary

13. Here, Planck's consideration suggests only that the energy of resonators received from the radiation to be discrete in nature. As I will show below, Jeans was first to suggest in 1911 that apart from energy in resonators being discrete, radiation's energy can vary only in discrete steps. Later, in 1913, Ehrenfest and Poincaré provided the clear proof for the same.

radiation field (Planck [1900b] 1967, 86). According to this procedure, R_0 is the probability of the equilibrium state of the system of resonators. However, there is nothing about the formulation of R_0 that makes it a probability notion, as generally understood. Planck was aware about this ad hoc stipulation of the notion. In the context where he defines this notion for the first time, he suggests that "we look for this distribution [i.e. R_0], if necessary by *trial* ..." (ibid., 86, emphasis added). Moreover, while discussing the soundness of his derivation[14], Planck revisits this stipulation about probability. Here, he comments that this consideration is "just a definition of the probability of the state, insofar as we have for energy radiation no other a priori way to define the probability than the definition of its entropy" (ibid., 87).

It is evident from the Planck's approach and his own comments that this notion of probability is merely an ad hoc stipulation. This notion of probability merely has a *theoretical* significance since R_0 can be considered as "probability" only when Planck adopts Boltzmann's approach and stipulates an expression in this position which will yield him the desired distribution equation. In other words, this conception of probability adopted by Planck has no grounding in the physical structure of radiation or matter and has no justification based on the nature of physical interaction between these two. Klein, in his analysis of Planck's derivation, observes the same and mentions that Planck's notion is "a probability *by definition*: he has no 'model' for understanding this probability in any sense analogous to Boltzmann's" (Klein 1961, 473, emphasis in the original). In contrast to Planck's, Boltzmann's invocation of probability has a clear justification. In Boltzmann's derivation, the state of the gas can be described as the composition of the states of its constituents. Since the system's state is constituted by its elements' state, the same specific state can be formulated in more than one way. These different "complexions" point to the same state of the system, but vary at the level of individuals since a molecule can be assigned any of the discrete energy values. With this, the "probability" of a state, W can be given a proper meaning: W is the ratio of the number of complexions for a given distribution of energy values to the total sum of all possible number of complexions for all suitable configurations of the system.

Style of Planck's Analysis

Planck's conceptualization of energy elements and probability definition highlights Planck's emphasis on the theoretical aspects while analysing the phenomenon at hand (radiation and matter interaction). Here, I am using "theoretical emphasis" to indicate not only the *mode of analysis* but also the *topic of enquiry*. It is evident from beginning that Planck is interested in deductively deriving a theoretical law from electromagnetic theory of radiation. Many years later, Planck describes the activity he carried out in the above

14. In Planck's words, "necessity of the above given *deduction*" (Planck [1900b] 1967, 87).

discussed two papers as an attempt to understand the "true physical meaning" of the distribution law (Klein 1961, 468). Nevertheless, it is clear from his work that "physical meaning" indicates theoretical clarification and not an attempt to provide the grounding for the distribution law through the physical structure of radiation and matter.

The emphasis on theoretical clarification is also evident through the idealizations of radiation and matter adopted by Planck in his analysis.[15] For instance, Planck considers harmonic oscillators as ideal representatives of matter "not because they were thought to be a realistic model for matter, but rather because Kirchhoff's theorem asserted that the equilibrium radiation distribution was independent of the system with which the radiation interacted, and oscillators were the simplest to treat" (ibid., 462). And with respect to radiation, throughout the analysis in these papers, the wave-interpretation, the then dominant physical model of radiation, played no significant role. Pointing out the absence of radiation's physicality in the analysis is not uncalled for. In contrast to the approach adopted in the above papers (of 1900), Planck's earlier attempts to ground the second law of thermodynamics in electromagnetic theory made explicit use of the physical-form of radiation. For instance, in a paper published in 1897, Planck analyses thermodynamic irreversibility as the "the alteration in the form of an electromagnetic wave by the scattering process — its apparently irreversible conversion from incident plane wave to outgoing spherical wave" (Klein 1985, 220). Here, in these earlier papers, Planck is categorical about the foundation of thermodynamics — it is not provided by statistical analysis, but should be grounded in the electromagnetic theory. However, Boltzmann responds to this and shows an inherent flaw in this approach. Planck finally realises the indispensability of statistical method and attempts to analyse radiation-matter interaction analogous to Boltzmann's treatment of ideal gas. In this revised mode of enquiry, radiation is no more waves having spherical or plane wave-fronts. Instead, Planck proposes the conception of *natural radiation*: "this was an assumption of randomness, of the absence of correlations between the various Fourier components of the radiation" (ibid., 221). This conception of radiation was Planck's attempt to introduce randomness in radiation which is analogous to "molecular chaos" present in Boltzmann's conception of ideal gases. This revision in Planck's analysis clearly highlights the shift in the mode of analysis — from radiation interpreted through its physical-form to the conception of it being constituted through various Fourier components. The following comment by Hendry captures the nature of Planck's analysis in 1900 papers aptly: regarding the adoption of Boltzmann's approach, Planck was clear that "use of this technique did not imply any acceptance of physical atomism ... and that its probabilities ... were merely those that *would* be required *if* the atomic assumptions were made" (1980, 61, emphasis in original). With this, it is clear that throughout Planck's analysis, the enquiry was neither about the physical form of radiation or matter nor any attention was paid to unravel the physical significance

15. For more about idealisation, see Weisberg (2007).

of the distribution law.

My intention of highlighting this should not be mistaken as a demand for the physical interpretation of Planck's new distribution law. Historically, after Planck's proposal, providing a coherent physical interpretation of this new distribution law was a formidable challenge. In fact, it is in the process of these attempts, that the proposal of wave-particle duality arises. One source of this problem was the final distribution law itself: at higher-frequency, one aspect of the law is applicable and at lower-frequency another aspect of the law becomes valid. Apart from this, another important source for this confusion regarding the physical interpretation of this law was the derivation provided by Planck. Planck's approach was patchy: in most places he follows Boltzmann's method which is contextualized in the wave-interpretation of radiation; however, at crucial places, he deviates from this approach. It is not clear whether Planck was aware of these deviations during the time of the initial proposal. The deviant probability calculation adopted by Planck was not new and was discussed by Boltzmann as illustration of an "non-physical example" (Hendry 1980, 62). However, as Hendry mentions in a footnote, Planck might not have been aware of this similarity between his method and the one discussed by Boltzmann. Even regarding the deviation of Planck from Boltzmann with respect to the handling of finite elements of energy, Klein suggests that Planck did not follow suit here due to his "apparent unawareness of the equipartition theorem and all it implied" (Klein 1961, 474).[16] Therefore, going by the above discussion, it can be concluded that Planck was interested in a particular kind of enquiry: the theoretical significance of his distribution theorem. The physical interpretation of the law or the physical significance of the phenomenon were never his concern. Therefore, with the emphasis on deriving a theoretical law, this whole analysis of Planck is shaped by theoretical and mathematical restrictions and this invariably comes at the cost of not having clarity about the physical interpretation of radiation.

Other Analyses

Even though by 1905, Einstein had proposed his light-quantum hypothesis, not many physicists took it seriously. Apart from Planck, other physicists too were cautious and chose to interpret the discrete energy only as an artefact of the theory. To highlight this, in this section two important physicists' views are discussed. One of the important contributions regarding the discontinuous energy variation of radiation was provided by J. H. Jeans in 1910. Even though Jeans was the first to provide a set of arguments in this line of enquiry, this paper had a limited impact during its time (Hendry 1980, 70). In this paper, Jeans attempts to examine the state of a system consisting of matter and ether through a set of physical laws that intrinsically respect continuity. According to this

16. Moreover, this deviation of Planck from Boltzmann regarding the elements of energy was recognized only in 1905 by Rayleigh and Jeans (Klein 1961, 474).

consideration, these physical laws are "expressible in terms of continuous motion (or of mathematical laws expressible in terms of differential equations)" (Jeans 1910, 944). The intention behind this examination, given the context of enquiry, is the following: would analysis of a system, through a generalized set of laws (which are continuous in nature), also result in a description of the equilibrium state (of the system) which respects Planck's law? If this is false, then it implies that an analysis (based on laws that respect continuity) will give rise to a description of the equilibrium state that is continuous in nature. However, if the description of the equilibrium state respects Planck's law, then it implies that any generalized analysis (based on laws that respect continuity) will eventually result in a description that does not respect continuity. Jeans, in this paper, shows that it is the latter which is true and hence the non-continuous nature represented by Planck's law is unavoidable. Jeans' argument considers the n-dimensional phase space of a system such that every point of this phase space is a possible state of the system. If the dynamic evolution of the system's state is considered as hydrodynamical steady motion, the average energy of the system respects equipartition theorem (ibid., 949). With this argument, Jeans concludes that "it is not enough to postulate systems of vibrators capable only of holding definite multiples of a fixed unit of energy . . . the energy in the aether itself must also be atomic" (ibid., 953).

Hence, Jeans provides the initial motivation that radiation's energy can vary in discrete steps. This argument of Jeans is carried out completely on theoretical grounds which makes use of the phase space of the system. Moreover, the analysis of the dynamic evolution of the system is carried out through theoretical idealization. Here, the idealization is achieved by using theoretical analogies like, for instance, considering the collection of points on phase space, each of which can be a possible state that the system can occupy, as "forming a continuous fluid" (ibid., 944). This comparison enables the use of other analogies for carrying out the theoretical analysis. The "hydrodynamical steady motion" of this fluid helps in the analysis of the dynamic evolution and the "mass" of the fluid provides "basis for the introduction of the calculus of probabilities" (ibid., 945). Recognizing that Jeans is dealing with the phenomenon of matter-ether interaction *theoretically* is important to realize that this analysis does not reveal anything about the *physical structure* of radiation explicitly. Jeans does not make these distinctions in his analysis. However, he goes as far as to suggest that "it is not sufficient that the energy should always in nature occur in complete atoms; what is required is that it should be physically impossible to divide these atoms. For instance, the requirements of this condition are not met by imagining a system of radiators which give off energy in complete units; we must also have an aether structure such that no vibrations can possibly exist in it except in atomic amounts" (ibid., 953). This is the only statement where it can be seen that Jeans thinks his theoretical analysis has some insights about the physical structure of the radiation.

The confusion regarding the interpretation of "elements of energy" and Einstein 1905 proposal about light-quantum continued for few more years before reaching a positive conclusion largely affected by the influence of Poincaré's paper in 1912. To these discussions, Paul Ehrenfest made a very important contribution, which was overlooked during that period. He was the first one to conceptually distinguish between the different connotations of "quantum" found in Planck's and Einstein's proposals. In 1905 itself, Ehrenfest had pointed that the hypothesis of light quantum should be taken "only formal in its present form ... it has no analogy in Boltzmann's theory" (Klein 1985, 234). He further developed his views in a 1911 German paper, whose title (in English)[17] — "Which features of the hypothesis of light quanta play an essential role in the theory of thermal radiation?" — brings out clearly that he is interested in understanding the meaning of "quantisation" at different levels. By comparing Planck's and Einstein's hypotheses, Ehrenfest shows that Planck's theory does not imply that radiation's energy is discrete (apart from matter-radiation interaction happening over discrete steps) and also does not propose that light-quanta have "separate existence in empty space" (ibid., 254). With this he concludes that *quantisation*, which is essential for Planck's theory, does not imply *corpuscularity* (Navarro and Pérez 2004, 133). In a 1914 paper, he along with Heike Kamerlingh Onnes further clarify that "More than once the analogous, equally formal device used by Planck, viz. distribution of P energy-elements over N resonators, has by a misunderstanding been given a physical interpretation, which is absolutely in conflict with Planck's radiation formula" (ibid., 133).

4.2.2 Einstein's Analysis of Light

It is important to discuss Planck's exploration of radiation before presenting Einstein's proposal. In the background of Planck's refusal to take his "elements of energy" seriously, Einstein's admittance of light-quantum as a genuine physical entity stands out clearly. However, as I will show, Einstein improvises the picture of radiation (as constituted of light-quanta) in 1909. The role of wave-particle duality concept played a central role in both these proposals of Einstein. It is this which will be stressed in the present section.

Einstein's Conception of Light in 1905

During the first decade of the twentieth century, electromagnetic interpretation was the prevailing conception of light. It is in the background of this context, that Einstein was about to propose his light-quantum hypothesis. In order to provide motivation for his proposal, Einstein starts his 1905 paper by pointing to the "profound formal distinction" between the theoretical foundations of light and matter. He points how these two theories differ in their approaches about conceptualisation of energy: if electromagnetic theory analyses energy through "continuous spatial function", in the case of ponderable entities,

17. The translation of the title is found in Klein (1985) and Navarro and Pérez (2004).

energy is calculated by adding the non-arbitrary contributions of the constituents parts (Einstein [1909] 1989, 367). By formulating the problem in this way, Einstein suggests that by considering light to be constituted of "finite number of energy quanta", the "dualism between particle and field", as Klein (1963, 62) labels it, can be resolved. In order to do this, Einstein first analyses a volume of monochromatic radiation using Wien's law (4.1). Since Wien's law is applicable only at high frequency range, Einstein is aware that his analysis is limited only to a specific situation (Einstein [1905a] 1965, 370). In order to set the ground for carrying out the statistical thermodynamical analysis of radiation, Einstein shows that the concerned volume of radiation is analogous to an ideal gas as both these systems behave similarly regarding their entropy variation over volume. With the similarity established, using Boltzmann's principle, Einstein concludes that "monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as though it consisted of a number of independent energy quanta..." (ibid., 372). These light-quanta are localised at specific point in space, indivisible and were always emitted or absorbed as "complete units" (ibid., 368). Thus, by drawing the analogy between radiation and ideal gas, Einstein showed that light-quantum is similar in characteristics to the thermodynamical entity constituting the ideal gas. Drawing this analogy was not straightforward as radiation does not behave similar to ideal gas at all conditions. Only at the higher frequency range and that too only in the context of random volume fluctuation process, radiation does possess the "macroscopic signature of a system of many spatially independent components" (Norton 2006, 88-91).

Thus, the 1905 paper of Einstein was the first step towards the physical interpretation of discrete "elements of energy" that were hesitantly presumed by Planck to make sense of the new radiation law. This transformation is also captured by the historical trajectory taken by the concept "quantum". This latin word was initial found in German scientific texts, an early instance of its usage is traced back to a 1883 manuscript of Boltzmann. Planck too uses "quantum" in his seminal papers but always in the context of referring to fundamental units of mass and electricity. So, he did not explicitly use this concept to refer to "elements of energy". However, in his 1906 book, Planck uses "quantum" to refer to the elementary quantity of action h. In the 1905 paper, Einstein uses "quantum" not only to refer to fundamental mass and charge, but also to refer to discrete light corpuscles (Klein 1985, 254). Post Einstein's usage, this concept came into usage both in English and French literature. In this sense, Einstein was primarily responsible for the reconfiguration of the concept by providing it physical connotation.

"Fusion" of Theories

After the 1905 proposal, Einstein's 1909 paper brought further development regarding the structure of radiation. In this paper, Einstein mentions at the beginning itself that "the next stage in the development of theoretical physics will bring us a theory of light that can be understood as a kind of fusion of the wave and emission theories of light" (Einstein [1909] 1989, 379). One of the central question that Einstein is interested in this paper, for which the above statement is supposed to be the response, is regarding the *constitution* of radiation. Einstein's conclusion regarding this enquiry is usually considered as the first articulation of wave-particle duality about light. In the initial part of the paper, Einstein outlines several arguments to indicate the weakness of undulatory theory of light and how in each of these scenarios, the corpuscular theory — radiation as constituted of small, independent corpuscles — explains the phenomena soundly. The wave model of light does not account for both the processes of emission and absorption of radiation adequately (ibid., 387). In this model, emission of radiation from an oscillating charge will be in the form of an expanding spherical wave. But the opposite process of absorption cannot be neatly accounted by the reverse picture where a spherical wave collapses on the charge. Moreover, by 1905 itself, Einstein had conjectured that radiation and its energy has to be focused and directed in a particular direction for photo-electron to gather all the energy impinged on it. This is contrast to the spherical expansion of the radiation where its energy is dissipated in all directions. Therefore, Einstein suggests, Newtonian emission theory does not face these problems since according to it radiation is emitted and absorbed as tiny particles.[18]

After having presented the series of arguments for corpuscularity, Einstein moves to the next stage of his argument by asking the following set of questions:

> *Planck's theory leads to the following conjecture . . . emission and absorption of radiation can take place only in quanta . . . The following question arises then . . . Would it not be possible to replace the hypothesis of light quanta by another assumption that would also fit the known phenomena? If it is necessary to modify the elements of the theory, would it not be possible to retain at least the equation for the propagation of radiation and conceive only the elementary processes of emission and absorption differently than they have been until now?*

It is indeed surprising that Einstein raises these queries after a prologue about the primacy of corpuscular theory over wave interpretation of light. The only way to make sense of this sequence of arguments is to interpret them in the context of his larger intention which is clarified at the beginning of the article itself. Therefore, the subsequent discussion that follows marks a significant turn in the enquiry of light. To answer the above questions, Einstein strategy is to start with the presumption that Planck's radiation law is correct and ask "whether some conclusion about the constitution of radiation can

18. This suggestions present in the 1909 paper was taken up further by Einstein in 1917 where, in the context of spontaneous emission and absorption of radiation by molecules, he argues that this process has to be directed in a specific direction (Klein 1964, 20).

be inferred from it" (Einstein [1909] 1989, 391). Einstein considers the following thought-experiment to analyse the queries. A cavity consisting of an ideal gas and a solid plate which can move in either directions perpendicular to its plane is considered. The cavity also contains radiation of frequency ν. Both the ideal gas molecules and the radiation impinge momentum on the plate. In this setup, if thermodynamic equilibrium has to be maintained within the cavity, the momentum fluctuation of the plate (Δ) should have the following expression:

$$\Delta^2 = \left(\frac{1}{c}\right)\left\{h\nu\rho + \left(\frac{c^3}{8\pi}\right)\left(\frac{\rho^2}{\nu^2}\right)\right\}f\tau d\nu \tag{4.4}$$

where, ρ is the radiation distribution function and h is the Planck's constant (ibid., 392). In the right hand side of the expression, Einstein recognizes two independent factors (present within the braces) responsible for the momentum fluctuation. The second term is the momentum caused by radiation waves which result from the interference process: "beams of not very different directions, not very different frequencies, and not very different states of polarization must interfere with each other, and to the totality of these interferences ... there must correspond a fluctuation of the radiation pressure". The first term, however, cannot be understood if light is considered as waves. This term, which becomes significant when the frequency is higher, indicates that radiation might be constituted of "very small-sized complexes of energy $h\nu$, moving through space independently of each other and reflected independently of each other — a conception that represents ... light quanta" (ibid., 393).[19] Thus, through this analysis, Einstein shows that radiation has unique behaviour signatures at different frequency ranges. At higher frequencies, radiation behaves like a system of particles and at lower frequencies, radiation can be understood through wave-picture. Since he thinks that "the two structural properties (the undulatory structure and the quantum structure) simultaneously displayed by radiation ... should not be considered as mutually incompatible," he proceeds further to provide the physical interpretation of radiation that can account for the schizophrenic behaviours of radiation (ibid., 394). According to this picture, radiation seems to be constituted by "singularities" of energy and each of these seems to be surrounded by "fields of force" (ibid., 394).

Duality of Radiation

Einstein, throughout his life, considered the 1909 proposal as the working model for radiation. For instance, in his correspondence with Lorentz in 1921, this picture of radiation is re-emphasised. And in 1926, Einstein and Emil Rupp's correspondence again revolves around the same interpretation (Dongen 2007, 123). However, Einstein was never

19. As Klein (1964, 10) points, this momentum fluctuation argument is not the only illustration that Einstein considers. Along with this, Einstein shows (in another paper) how derivation of energy fluctuation based on similar principles exhibits dual components.

happy with this model of radiation and accepted it only as a tentative hypothesis. Because of this, Kojevnikov observes that Einstein accepted this interpretation “only in a negative sense, as a fundamental difficulty of quantum theory that had to be resolved rather than turned into a postulate” (Kojevnikov 2002, 217). Nonetheless, the impact of the changes that Einstein brought regarding the physical interpretation of radiation within a span of five years for the subsequent analyses is yet to be understood and appreciated.[20] Even if we focuses only on Einstein’s own views in 1905 and 1909, the shift in the conception of light are quite remarkable. In 1905, Einstein limits his analysis to higher frequency range and with that concludes that radiation is particle-like. In 1909, he attempts to address this narrow analysis and makes the bold move to understand radiation across the entire range. Here, he realises that light cannot only be understood through its corpuscularity. If its behaviour at lower frequencies have to be accounted, light needs to be interpreted as waves. Because of this realisation, he does not completely rejects his 1905 proposal, but improvises it.

This significant shift in Einstein’s views has not been given its due importance. And this has resulted in multiple interpretations of Einstein’s views, the most controversial one among these being the “wave-particle duality”. Most of the surveys about duality or quantum mechanics — like for instance Klein (1964), Jammer (1966, 37), Hendry (1980) and Pais (1986, 248) — consider the hypothesis proposed by Einstein in 1909 as the first formulation of wave-particle duality. However, as mentioned in (4.1.4), Einstein never invoked “duality” in his paper. Instead, the concept he preferred was that of “fusion” of theories. Even Kojevnikov exhibits caution about labelling this proposal as “duality” formulation. For him, this fluctuation formula could be interpreted in variety of ways, “*duality being only one of them*” (Kojevnikov 2002, 183). For him, the establishment of a “permanent link in popular perception between fluctuations and duality” was a “historical contingency” that resulted with several authors becoming aware of the fluctuation analysis present in Einstein’s papers and attempting to resolve the problem presented by Einstein (ibid., 217-18). Thus, the retrospective attribution of duality to Einstein’s 1909 analysis has resulted in several confusions. Given the complexity of Einstein’s analysis and its troubled relation with Einstein’s 1905 views, it is not clear where to situate “duality”. Is this reference to Einstein pointing out the dual behaviours of radiation at two different frequency ranges? Or “duality” indicates the contrived physical structure of energy singularities surrounded by ghost fields that Einstein hesitantly proposes?

Klein (1964, 15), for instance, considers the suggestion of “two independent mechanisms” as Einstein’s “wave-particle features of radiation”. Here, Klein does not further qualify what he means by “mechanisms”. Duncan and Janssen (2008) provide a much more

20. Given the influence of the momentum fluctuation analysis had on the theoretical development in the later decades, Darrigol (1986, 199) has labelled it as the “most famous and universal puzzle of radiation theory”.

clearer but stronger interpretation of Einstein's wave-particle duality. They base their analysis of Einstein's fluctuation formula on Klein's interpretation (stated previously) and emphasise that Einstein's claim is about the presence of two different processes. However, as they proceed further, their interpretation of Einstein's duality becomes crisper. Einstein, according to them, is suggesting that the two independent components of the momentum fluctuation are arising from two different things: waves and light-quanta (Duncan and Janssen 2008, 638). Regarding this, the authors are interested in showing that Einstein's interpretation of the fluctuation is incorrect since it presumes the existence of two things — waves and quanta. Contrast to that, they set out to argue that Pascual Jordan's work (in a paper written in 1926 with Born and Heisenberg) "shows, contrary to Einstein's expectation, that both terms in the fluctuation formula can be accounted for within a unified dynamical framework" by considering the quantised wave alone as the starting point (2008, 635). Given this, the authors seem to be suggesting that Einstein's proposal of "duality" is about radiation consisting of two different kinds of entities — waves and light-quanta. Even though in the Einstein's paper, there are some elements which support this interpretation, as mentioned, given that Einstein does not propose "duality", it does not make sense to ask further what he actually meant by it.

Irrespective of what specific aspect of Einstein's analysis is considered as "wave-particle duality" proposal, Einstein's struggle, both in 1905 and 1909, were regarding the physical interpretation of light. And this interpretation was largely guided by Planck's law. Planck himself spent enormous effort in grappling with the concept of "elements of energy". The later birth of light-quantum and the subsequent addition of undulatory behaviour to light by Einstein were all centred around the important developments, which can be loosely labelled as "duality".

4.3 Applicability of Classical Concepts for Light

In the initial quarter of the twentieth century, Einstein's proposals were not the only ones that were upsetting the classical understanding of light. There was a steady increase in the recognition that wave-picture alone will not suffice to describe light's behaviour. In this scenario, there was no clear consensus among the physics community. Einstein and few others felt that future clarifications about the physical structure of light will resolve this tension. Others like Heisenberg and Dirac considered that the resolution lies in finding the right theoretical interpretations. In contrast to these, Bohr had a completely different opinion. The prominent shift that affected Bohr's views was with respect to the reality of light-quantum. Much throughout the initial two decades after its proposal, Bohr argued against the light-quantum concept. With the failure of Bohr, Kramers and Slater's proposal, Bohr realised the inevitable need of corpuscular interpretation of light (Jammer 1966, 183-86). These developments and various other aspects shaped Bohr's unique response that the "wave" and "particle" behaviours of light (and matter) should be understood as *complementary* pictures of the same reality. Of course, as Beller (2001, 225) mentions, Bohr responded with complementarity principle because he was, unlike others, was primarily preoccupied with the paradoxical experimental observations. Another broader reason for the rise of this unique proposal was the worldview that Bohr, Heisenberg and others subscribed to, which was much different than Einstein's.

Einstein was greatly influenced from the rivalry between the mechanical and electromagnetic worldviews prevalent during his time. As already noted in (2.1.3), by thes late nineteenth century, physicists were against the mechanical interpretations of phenomena. As McCormmach (1970a, 461) notes, since most of the German physicists during this time held that "the present object of electromagnetic theory should not be to penetrate the mechanism, but to find the simplest equations for describing the phenomena", the trend towards abstract theories guided by mathematical principles had started. In this milieu, even though Einstein's style was influenced by both the worldviews, it was very distinct from these. Being greatly influenced by Hume and Mach, Einstein was opposed to the mechanistic interpretations (Hirosige 1976, 61). But this does not imply that Einstein favoured the electromagnetic worldview. By rejecting the electromagnetic ether, Einstein deviated from the electron theory of Lorentz (McCormmach 1970b, 53). Einstein, instead of showing the supremacy of one over the other, was motivated towards unifying these two worldviews (Hirosige 1976, 74). His energy-mass equivalence proposal exemplifies clearly this desire (McCormmach 1970b, 65). The unique style of Einstein also characterises his interpretation of radiation: even though he embraces the development of electromagnetic theory and Planck's discovery of radiation law, he still argues for the necessity of understanding the physical significance of these new theoretical developments. Einstein was not alone in thinking that new worldview is necessary for understanding the modern developments. Several physicists during the beginning of the twentieth century felt that the quantum revolution brought by Planck's law conflicted with the electromagnetic worldview (Seth 2004; McCormmach 1970a, 488). Thus, the rise of quantum mechanics along with theory of relativity were responsible for dethroning the electromagnetic worldview (McCormmach 1970a, 496). Bohr, Heisenberg and others like Dirac and Jordan belonged to this new worldview that eschewed mechanical concepts and embraced mathematical, abstract thinking (de Regt 2017; Camilleri 2007a). Even though, these physicists differed from one another with regard to the key concepts in quantum mechanics, most of them shared the belief that classical concepts were not sufficient for quantum mechanics. Because of this, Bohr, Heisenberg and others provided different interpretations of light in the context of wave-particle duality.

Bohr understood radiation in the context of wave-particle duality through the principle of complementarity. This principle provides a way of making sense of the relation between pairs of classical concepts like wave and particle descriptions, spacetime and causal descriptions, momentum and velocity characteristics. In the 1927 Como lecture, where Bohr articulates this principle, his main concern was about the "fundamental limitation" of the classical physical ideas for atomic phenomena. To highlight this, he shows how in the quantum theory, the "space-time co-ordination" and the "claim of causality" turn out to be "complementary but exclusive features of the description" (Bohr 1928, 580). With the help of these case studies Bohr intends to portray the "fundamental limitation in the classical physical ideas when applied to atomic phenomena" (ibid., 580). The complementarity principle helps in overcoming this limitation of classical concepts and thus resolve the paradoxical situations which arises while describing various quantum mechanical experiments (ibid., 582).[21] Even though in his 1927 lecture, Bohr does not explicitly discuss about duality, his discussions with other physicists during that time clearly bring his views of the wave-particle complementarity. For instance, in his response to Heisenberg's 1927 paper on uncertainty principle, Bohr uses the gamma-ray thought experiment to illustrate how the wave and particle interpretations of light are mutually exclusive (Beller 1992; Camilleri 2006).[22].

4.4 Equivalence of Wave and Particle Interpretations

Another thread of development regarding wave-particle duality and analysis of radiation was regarding the equivalence between two theoretical formulations, proposed by Heisenberg. Right from the very beginning, Heisenberg was of the opinion that classical concepts were unfit for quantum mechanics. However, this opinion of his underwent several revisions. Heisenberg was influenced by Pauli's view to eschew mechanical approach and embrace abstract mathematical worldview to do physics (de Regt 2017, 236). Being also inspired by Einstein's revolutionary contributions to theory of relativity, Heisenberg considered that concepts like space and time in the context of quantum mechanics cannot bear their classical meanings. For instance, in 1925, with regard to the difficulties of interpreting "wave" and "particle" behaviours of light and matter, he was of the opinion that space and time have to be discontinuous (Camilleri 2007a, 182). He revised his stance in 1927. Under the influence of the operationalism interpretation of science, he considered that physical concepts can be provided meaning only in the context of specific experiments. It is while putting this philosophy into action, Heisenberg proposes the well known thought experiment of the gamma-ray microscope in order to arrive at a precise operational meaning of "position" and "velocity" for quantum mechanics (ibid., 188). Bohr, while responding to these thoughts of Heisenberg, argues that the right way to interpret these results and

21. Bohr, as Bala (2016) claims, later in his life used Eastern philosophies like Buddhism to interpret the complementarity principle. Bala also discusses the conceptual influence this principle had on other disciplines like psychology, biology and anthropology.

22. A detailed study of Bohr's complementarity and its application for wave-particle duality will be further carried out in the section 5.3

the uncertainty principle is within the complementarity framework. However, in 1929, Heisenberg revises his views on duality yet again. As already mentioned in section 4.1.2, Heisenberg articulates the theoretical equivalence between particle and wave pictures. Deviating from Bohr, Heisenberg argues that both "wave" and "particle" interpretations are not necessary and one is free to chose either of the language to describe the phenomenon at hand. And more importantly, Heisenberg's interpretation of duality was situated in the context of theoretical analysis, unlike Bohr's view that pertained to the description of experimental observations (Camilleri 2006, 309). When this conclusion of Heisenberg is placed in the series of revisions his worldview underwent, it can be clearly seen how the final view on duality is the matured version of his earlier thoughts.

It was the works of Dirac, Jordan and others during the period of 1927-28 that motivated Heisenberg to change his view about duality. Therefore, to understand what he means by theoretical equivalence, it is necessary to consider Dirac's remarkable contribution. In his 1927 paper, Dirac interprets radiation in the quantum mechanical framework. He starts by considering a radiation field within an enclosure such that each of its Fourier components, say r, has energy E_r and phase θ_r. These two variables of each component can be considered as canonically conjugate variables and the canonical equations of these variables can be used to calculate the Hamiltonian of the whole field. Till this point, the analysis carried out is similar to the classical approach. Where the quantum mechanics analysis differs is how it interprets these canonical variables. Under this non-classical approach, E_r and θ_r are no more standard variables to which certain number can be associated; instead these dynamic variables are matrices which are non-commutative under multiplication: $\theta_r E_r - E_r \theta_r = i\hbar$ (Dirac 1927, 243). The conjugate variables respecting the "standard quantum condition" implies that the energy associated with each component of the radiation can vary only in discrete quantities.[23] Thus, quantum mechanical interpretation "immediately gives light-quantum properties to the radiation" (ibid., 244-45). Here, at this particular stage, given the context of the paper, Dirac's use of "light-quantum" refers only to energy of r being quantised and suggests that the energy can vary only in discrete steps.

Regarding the nature of light-quantum, Dirac has a unique proposal. He does show that radiation can be interpreted as constituted by light-quanta. However, this does not imply his theory promotes the physical reality of these entities. Instead, he shows that the quantum theoretical interpretation of radiation does not essentially depend on the physicality of radiation. By showing that both the physical interpretations of radiation (radiation as a series of waves and as a stream of particles) provide the same expression for the hamiltonian of the system, he argues that "the wave point of view is thus consistent with the light-quantum point of view" (ibid., 263). The equivalence of both the interpretations is based on the unique consideration of quantum theory where

23. This is because for a particular frequency ν_r of the component r, since the phase is an angle given by $2\pi\nu_r\theta_r$, the energy (which will be $E_r/2\pi\nu_r$) can vary only discretely.

E_r and θ_r are considered to be non-commutative conjugate variables. In order to show the equivalence, Dirac first analyses the Hamiltonian of an assembly of systems (which is radiation) and perturbing system (which is an atom) using the quantum theory approach. This approach, with the application of Schrödinger's equation, provides the probability of radiation being in each of the component's state. The Hamiltonian thus formulated, which provides the general description of the interacting systems, is what Dirac considers and shows that when radiation is interpreted either as an assembly of light-quanta or as constituted by several Fourier components, the description of the system comes out similar.

Enrico Fermi, in a series of lecture delivered in 1932, summarises Dirac's theory in the following way: "until a few years ago it had been impossible to construct a theory of radiation which could account satisfactorily both for interference phenomena and the phenomena of emission and absorption of light by matter . . . Dirac succeeded in constructing a quantum theory of radiation which could explain in an unified way both types of phenomena" (Fermi 1932, 87). As this remark by Fermi convey, prior to Dirac's theory, wave and particle interpretations of radiation were considered as different interpretations. With the hamiltonian of both the interpretations being shown to be similar, the new theory changes the relation between these two interpretations: these views are no more competing interpretations; instead they are merely alternative descriptions which have equal legitimacy. Given this equivalence, depending on the situation and need, suitable interpretation of radiation can be used. This approach can be illustrated through the case studies provided by Fermi in his review article. He discusses few traditional wave phenomena — like the propagation of light in vacuum and Lippman interference fringes — and few particle phenomena — like Doppler effect and Compton scattering — and shows how both these sets of phenomena can be explained by the new quantum theory of radiation.

It follows from Fermi's discussion that the new theory of radiation does not necessarily recommend a particular physical interpretation of radiation. This does not mean that the theory does not require any model for interpreting the phenomena. The situation is completely the opposite - the theory works for not one, but for both the models. In the paper, Dirac seems to be interested in implementing his "general transformation theory of the quantum matrices" to understand interaction between electromagnetic field and matter. To this end, he develops the theory of interaction without presuming any structure of radiation except for stipulating that radiation should satisfy Einstein-Bose statistics (Dirac 1927, 256). With this general theory established, a brief note on the difference between Jordan's and Dirac's notions of wave and particle interpretations since that clearly brings out the difference in the notion of light-quanta each of them provides. Kragh mentions that Jordan's main aim was to prioritise "(quantum) waves over particles" and "derive the corpuscular properties of radiation . . . in terms of the quantization conditions imposed on wave-propagating quantum fields" (Kragh 1990, 130). Thus, for Jordan, the notion of light-quanta is merely a property of electromagnetic waves. Contrast to that, for Dirac, as already mentioned in section 4.1.3, the wave and particle interpretations are "just two mathematical descriptions of the same physical reality" (ibid., 338n40). In this sense, Dirac holds that both the models are of equal status.

Chapter 5

Critique of Wave-Particle Duality of Single Photons

In the previous chapter, I highlighted the varied meanings of wave-particle duality that are formulated at the various stages of doing science: while interpreting experimental observations, at the level of theorizing and during the evaluation of the ontology of the physical world. These notions of duality differ from one another not only about the context of formulation, but also with respect to the meanings of "wave", "particle" and "duality". Among these multiple connotations, the present chapter is concerned with the duality claims about single-photons that are situated in the context of two experiments: the anti-correlation demonstration of photons and the Mach-Zehnder interference experiment. Articulated with the help of the complementarity principle and specific analogies, this type of duality claims is categorically distinct from other formulations and is firmly situated in few experiments. In this chapter, I want to highlight the weakness of these claims by arguing that the respective analogies, on which the claims stand, cannot be formulated distinctly and strongly. A similar claim has been proposed by Bunge (1968). He argues that the use of analogies in physics — especially "classical-wave" and "classical-particle" in quantum mechanics — have serious limitations and they should be avoided. And since QED provides a better way to talk about phenomena without the need of analogies, Bunge concludes that duality is "a relic of 1905-1927 interregnum" (ibid., 271). However, as I will show in detail later, Bunge's argument is not enough to understand the weakness of the concerned duality.

I begin this chapter with a brief summary about how quantum optics interprets photons' behaviour in section 5.1. After this, I summarise the above mentioned experiments and provide the technical analysis of them. Subsequently, I mention few instances of the duality claims about photons in section 5.2. Here, I provide a detailed analysis of the duality found in these claims and bring out its salient features by comparing it with the other notions of duality. While doing this, I recognise two distinct ways in which duality of photons has been articulated: the claims either illustrate the wave and particle behaviour of photons in the concerned experiments or they use the principle of complementarity to further substantiate the duality. Among these versions, there are already several criticisms about the duality inferred through the complementarity principle. I summarise them in the section 5.3. With this version rejected, the motivation for duality completely resides on the possibility of showing that single-photons are "wave-like" and "particle-like" in the concerned experiments. It is this type of claim that I want criticise. In order to do so, I will first show in section 5.4 that the meaning of these analogies needs to be understood in the specific context of invocation and then sketch the general process for evaluating them. In sections 5.5 and 5.6, I carry out the analysis of particle and wave analogies in their respective scenarios. In each of these, I argue that all possible ways of establishing the similarity between photons and waves/particles are either unconvincing or problematic. Thus, by showing that the constitutive analogies are not clearly determined, I conclude that the duality about single-photons turns out to be a weak claim.

5.1 Quantum Optical Analysis of Photons

5.1.1 Difference between Classical and Quantum Optics

The experimental conclusion drawn by Grangier et al. (1986) is often considered to be the first accurate demonstration of wave-particle duality in the case of single-photons. Based on the behaviour of light in single-photon state in anti-correlation experiment and in interferometer experiment, they concluded that single-photons behave both "particle-like" and "wave-like". Prior to this demonstration, there have been several experiments that attempted similar kind of study. But the novelty of the experiments carried out by Grangier et al. was with regard to the theory and the experimental procedure adopted to achieve the presence of a single-photon at a given time. All the previous experiments were contextualised in the classical or the semi-classical electromagnetic theories of light. In these theories and in the respective experiments, the presence of single-photons is confirmed based on the rate of energy-flux and the appearance of discrete detection counts. In spite of these procedural techniques, since the experiments used chaotic light sources, it cannot be said that the recorded observations strictly correspond to single-photons' behaviour. Moreover, the notions of photon found in these theories are either naive or completely redundant for explaining the observations (Grangier et al. 1986, 174; Aspect and Grangier 1987, 4-5). Because of these reasons, the results of the previous experiments were considered to be inconclusive. Two important developments took place post 1950, that enabled Grangier et al. (1986) and others to overcome these hurdles. First was the technological feasibility of producing and detecting highly sensitive light beams. For instance, it was possible to produce radiation in single-photon state (henceforth referred to as SPSR). This revolution paved the way for the second important development — the advent of quantum optics which provided a sound theoretical conception of photons.[1]

Classical optics could not have provided the correct analysis of photons since it was capable of describing only two kinds of light: chaotic and coherent. These two kinds were distinguished either based on the source's photon-number fluctuation or through the light beam's coherence properties. Depending on the range of the values for each of these properties, classical optics labelled the light as coherent or chaotic. In 1960s, the exercise of reinterpreting optics using quantum mechanics and QED indicated that the above ranges are not the only modes within which light can operate. Light beams can exhibit degrees of coherence and photon-number fluctuation values that were not prescribed by classical optics (Loudon 2001, 180). Quantum optics, in contrast to the classical scheme, provided a tripartite classification of light: chaotic, coherent and non-classical light. These types of light, when expressed by the range of photon-number fluctuation values, are labelled as *super-poissonian*, *poissonian* and *sub-poissonian* light respectively (ibid., 199). On the other hand, the analysis of coherence properties categorises these as *bunched*, *coherent* and *antibunched* light (ibid., 250). The discovery of the non-classical properties of light and the theoretical interpretation of these by quantum optics were foundational for working with photons. Therefore, only by using a non-classical source[2] that generates light in single-photon state and whose emission rates are such that the photons are "not bunched", Grangier et al. (1986) could ascertain that the experimental data were produced by single-photons.[3]

5.1.2 Experiments Concerning Duality of Photons

Grangier et al. (1986) and subsequently others have articulated the duality claim about photons in the context of two experiments. To understand these claims, the general structure and outcome of these experiments is essential. The first one is a simple anti-correlation

1. For a historical account about the intimate relation between the invention of laser and the development of quantum optics, see Bromberg (2016).

2. For more on single-photon sources, see Grangier and Abram (2003) and Lounis and Orrit (2005).

3. This discussion on the generation of single-photons and execution of highly specific experiments on them gives the impression that quantum optics established a firm realistic conception of photons. Even though the modern theoretical changes helped in abandoning the naive presuppositions of the classical optics, it is not true that these changes resulted in a clear precise concept of photons. Contrary to that, the notion of photon became even more vague. In 1960s, the situation worsened to such a degree that some physicists proposed a ten-year moratorium on the usage of "photon" (Hentschel 2018, 149), while others insisted that "a license be required for use of the word 'photon'" (Lamb 1995). The confusion about photons has further aggravated in the recent times since their ontological status is not guaranteed in QFT (see Halvorson and Clifton (2002)). All these open questions highlight that caution should be exercised while talking about photons and experimental observations cannot be simplistically interpreted as "produced by photons". Having acknowledged these difficulties, the discussions about photons in the current chapter pertain to certain papers where the notion of photon is contextualised in specific experimental scenarios. For more on the distinction between theoretical and operational conceptions of photons, see Loudon (2001, 1-2) and Scully and Zubairy (2001, 28-35).

experiment (henceforth AC) that illustrates an important non-classical characteristic of light. As figure (5.1) shows, in AC, the light emitted from a source is directed towards a beam-splitter (B_{ac}) that has two detectors — $D1_{ac}$ and $D2_{ac}$ — on either side of it. When a regular-light source (chaotic or coherent) is used, the light beam splits equally at B_{ac} and both the detectors simultaneously record the same number of photons. However, when an antibunched-light source is used, at any given time only one detector is actuated. The difference between these two observations can be better expressed through the status of $D1_{ac}$ and $D2_{ac}$. In the first case, since both the detectors are actuated, they are said to be in "correlation" with each other. In the second case, the detectors are always "anti-correlated": the actuation of a detector implies that the other one will not get activated simultaneously.

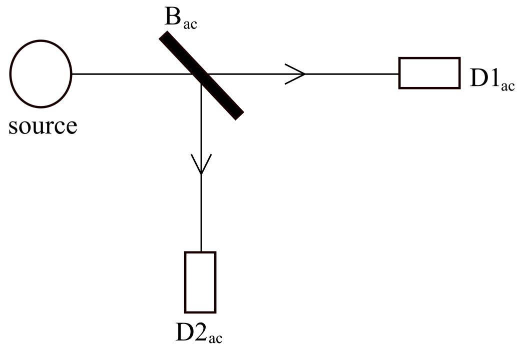

Figure 5.1: Schematic representation of AC

The anti-correlation observation can be explained by the characteristic photon distribution in antibunched-light beam. With single-photon state emissions being evenly spaced out in time, at any instant only one single-photon state pulse arrives at B_{ac}. Since quantum mechanics suggests that "a single photon can only be detected once", either of the detectors and never both get actuated (Grangier et al. 1986, 173). This deviancy in the behaviour of photons from classical light cannot be accounted in the classical field theory (henceforth CFT). Thus, AC with antibunched-light brings to the forefront a fundamental non-classical characteristic of photon that is otherwise not possible to observe in other scenarios.

Grangier et al. (1986) also studied the behaviour of single-photons in Mach-Zehnder interferometer (henceforth MZ). MZ, as shown in figure (5.2), consists of a beam-splitter ($B1_{mz}$) at the source and another one ($B2_{mz}$) just prior to the photon detectors. There are two detectors $D1_{mz}$ and $D2_{mz}$ on either side of $B2_{mz}$. Light, after passing through $B1_{mz}$, can traverse either the upper path or the lower one to reach $B2_{mz}$. Classically understood, the interference phenomenon here is between the beams coming from the upper and the lower paths and the location of interference is $B2_{mz}$. Depending on the path-difference between the upper and the lower paths, constructive interference pattern is observed in one detector, say $D1_{mz}$, and destructive interference pattern in the other, say $D2_{mz}$.

MZ is usually carried with coherent light source. When this experiment is carried out with an antibunched-light source, an interesting observation is made. As expected, single-photons are always detected at either of the detectors, but never both. However, in contrast to the observation in AC, where the probability of photons going to either of the detectors is the same, in MZ the probability of photons going to a specific detector depends on the path-difference between the arms. For a given configuration, photons are always received at the detector configured for constructive interference, say $D1_{mz}$, and no photons are received at the destructive interference detector $D2_{mz}$. This observation is usually interpreted as interference exhibited by single-photons.

5.1.3 Analysis of AC

The common aspect in both the above mentioned experiments is the presence of beam-splitter (BS). In AC, light from a source goes past a BS and gets detected in one of the two photodetectors placed on either side of BS. In MZ, there are two BS such that the light getting transmitted and reflected from the first is made to go through the other. Therefore, in both these experiments, it the presence of BS that brings out the characteristic phenomena of radiation. Given this, it is important to understand radiation's behaviour when it passes through a BS.

In figure (5.3) the schematic representation of BS as depicted in Loudon (2001, 89) is shown. Here, E represents the electric fields at the respective faces of the BS. The light enters BS from either of the two sides — E_1 and E_2 — and the input electromagnetic field either gets transmitted or reflected. So, for E_1, E_3 is the reflected light and E_4 represents the transmitted light. Given this direct relation between the input and output fields, each of the output field can be expressed in terms of the input fields:

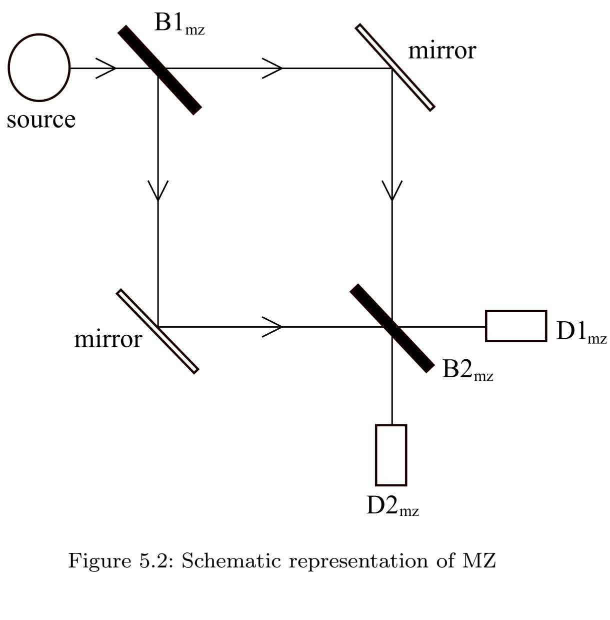

Figure 5.2: Schematic representation of MZ

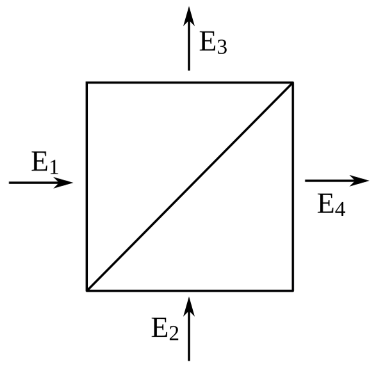

Figure 5.3: Schematic representation of BS

$$
\begin{aligned}
E_3 &= R_{31}E_1 + T_{32}E_2 \\
E_4 &= T_{41}E_1 + R_{42}E_2
\end{aligned}
\tag{5.1}
$$

In these equations, R and T are beam-splitter coefficients: transmission and reflection coefficients respectively. These coefficients have two factors — amplitude and phase-factors (Loudon 2001, 90). Therefore, in order to articulate the behaviour of light transmitting through BS, it is important to understand the nature of these coefficients and the relation between them. Given that the phenomena of reflection and transmission through BS respect principle of energy conservation, it is possible to find the properties of these coefficients. Without going into the details of this derivation, the important characteristics of these coefficients for a symmetrical beam-splitter are:

$$
\begin{aligned}
|R|^2 + |T|^2 &= 1 \\
\phi_R - \phi_T &= \pm\pi/2
\end{aligned}
\tag{5.2}
$$

where $|R| \equiv |R_{31}| = |R_{42}|$ and $|T| \equiv |T_{32}| = |T_{41}|$ and ϕ are respective phase angles (ibid., 91).

The above analysis of the coefficients of beam-splitter was carried out in the classical field theory framework. The relation between these coefficients stay intact even in the context of QFT, where fields are described through field operators, like construction and destruction operators (ibid., 214). These operators are related to one another through commutation relation. In QFT analysis, the input and output fields across the four sides of BS are represented by field destruction operators — $\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4$. Like the relation between the output and input fields mentioned above for classical scenario, these field operators are related in the same way:

$$
\begin{aligned}
\hat{a}_3 &= R\hat{a}_1 + T\hat{a}_2 \\
\hat{a}_4 &= T\hat{a}_1 + R\hat{a}_2
\end{aligned} \tag{5.3}
$$

The input destruction operators can be expressed in terms of the output operators:

$$
\begin{aligned}
\hat{a}_1 &= R\hat{a}_3^\dagger + T\hat{a}_4^\dagger \\
\hat{a}_2 &= T\hat{a}_3^\dagger + R\hat{a}_4^\dagger
\end{aligned} \tag{5.4}
$$

where, $\hat{a}^\dagger$ represents respective construction operator (Loudon 2001, 214).

With these basic relations in hand, important characteristics of SPSRs in the context of BS could be articulated. Consider the situation where one of the input beams, say $\hat{a}_1$, is radiation having single photon, represented by its photon-number $|1\rangle$. The other input beam is in the vacuum-state $|0\rangle$. Since this whole analysis of input and output beams' relation is carried out with principle of energy-conservation as the starting point, it has to be the case that the photon-number between input and output beams of BS has to be conserved (ibid., 216). Given that radiation of single-photon energy enters the BS, the output can have only $|1\rangle$. The important question is how does the single-photon energy gets distributed since the SPSR can get partly reflected and transmitted. This can be found out using the relations that connects the input and output operators mentioned above:

$$
|1\rangle_1 |0\rangle_2 = R|1\rangle_3 |0\rangle_4 + T|0\rangle_3 |1\rangle_4 \tag{5.5}
$$

5.1.4 Analysis of MZ

The above analysis of the coefficients of BS can be readily applied for interpreting MZ. The schematic diagram of MZ is provided in the figure (5.4). This experiment consists of two beam-splitters such that the light is passed through one of the input arms of the first BS, say E_1 and the output beams from the first is passed on to the second one by making the light-beams pass through path z_1 and z_2. These paths usually have different lengths and the path difference between these two paths plays the central role in the interference phenomenon observed in one of the output arms of the second BS.

In this setup, the electric field E_4 can be expressed in terms of the coefficients as

$$
E_4(t) = RTE(t_1) + TRE(t_2)
$$

where t_1 and t_2 are the delay in the light-beams in z_1 and z_2 because of the path difference between them (ibid., 92). The intensity of light at arm 4, when calculated as mentioned in the above manner, consists of not only the two components of radiation

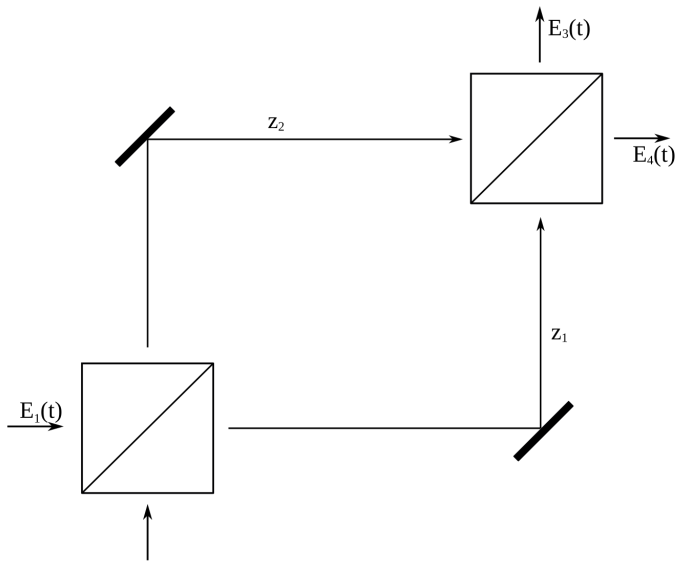

Figure 5.4: Schematic diagram of beam-splitters in Mach-Zehnder Interferometer

present in the two arms, but also a third component which is a combination of the light-beams arising especially due to their path-difference (Loudon 2001, 92).

The analysis of MZ in QFT remains similar to the analysis in classical framework apart from the prominent difference of expressing the input and output field components in terms of operators. As in the case of AC, here too, it is possible to express the two output destruction operators in terms of the input operators:

$$\begin{aligned}\hat{a}_3 &= R_{MZ}\hat{a}_1 + T_{MZ}\hat{a}_2 \\ \hat{a}_4 &= T_{MZ}\hat{a}_1 + R'_{MZ}\hat{a}_2\end{aligned} \tag{5.6}$$

where R_{MZ}, R'_{MZ} and T_{MZ} are composite reflection and transmission coefficients (ibid., 219). These composite coefficients are the outcome of the presence of two beam-splitters having the same reflection and transmission coefficients. However, the composite coefficients are not mere product of individual coefficients since the path difference between the beam-splitters plays a crucial role. The path-difference appears as phase factor in the expressions of these coefficients as shown below —

$$\begin{aligned}R_{MZ} &= R^2 e^{ikz_1} + T^2 e^{ikz_2} \\ R'_{MZ} &= T^2 e^{ikz_1} + R^2 e^{ikz_2} \\ T_{MZ} &= RT(e^{ikz_1} + e^{ikz_2})\end{aligned} \tag{5.7}$$

With these coefficients in hand, it is possible to arrive at the average photon-number count at the arm 4 (ibid., 220) —

$$\langle n_4 \rangle = |T_{MZ}|^2 = 4|R|^2|T|^2\cos^2[\frac{1}{2}k(z_1 - z_2)]$$

As the above equation shows, the average count at arm 4 for MZ is very different from the one for a single beam-splitter, which is equal to $|T|^2$. This equation reveals clearly how average photon count now is a function of path-difference.

5.1.5 Summary of Analysis

I have described in detail the theoretical considerations behind AC and MZ in the above subsections. I have showed specifically how the analysis of beam-splitter's reflection and transmission coefficients are fundamental for predicting the experimental results. The foundational premise behind the whole analysis was the conservation of energy. Along with this principle, if commutation rules of quantum field operators are taken into consideration, then it is possible to account for the observations of the respective experiments. In the present section, I will continue this analysis further. Specifically, I analyse what these two experiments tell about SPSR as a physical entity.

As mentioned, the field consists of two components — the amplitude component and the phase component. As I will show, both the experiments will bring out each of these characteristics of the field. The anti-correlation experiment consists of a single BS and two detectors on either side of the output arms. As already described, when a single SPSR is fed to one of the input arms, the theory suggests that the single energy-quantum has to be present in either of the arms and never in both. Since the phase-shift of the input SPSR is not much of a concern for a symmetric beam-splitter, this experiment then brings out the unique characteristic of the energy-quantum of SPSR (i.e., the amplitude of the field). AC, therefore, appears as a simple straightforward experiment that brings out the characteristic of the "quantum" feature of SPSR's energy.

The interferometer experiment consists of two beam-splitters and the final output arms' coefficients are composite in nature. As I have already shown, the average photodetection number at output arms is not just a function of the input values, but also of the path-difference. Because of the dependency of the output photon-number on the path-difference, the photon-count varies with the path-difference. This results in the interference-fringe-like pattern at the one of the output arms. Here, to clarify, the variation is of the photodetection for a certain period of time. So as the path-difference varies, the total photodetection number for a particular period either reduces or increases. Because of the observation of this interference-like pattern, MZ is considered to bring out the wave-like behaviour of SPSRs.

5.2 Duality Claims about Photons

With respect to the unique traits of single-photons in AC and MZ, some analyses have claimed that these experiments bring out their “wave-like” and “particle-like” behaviours. The duality of single-photons have been argued largely in two ways. In the first version, the principle of complementarity is used to substantiate the presence of duality. In the other approach, duality emerges when the observations are interpreted using certain analogies. To understand the first version, consider the quintessential behaviours of photons in the concerned experiments. In AC, single-photons, after passing through B_{ac}, have equal probability of ending up on either of the detectors. When this setup is modified to form MZ by adding another beam-splitter, the probability of photon-detection at the detectors completely changes. This difference in observations makes it difficult to come up with a coherent interpretation of AC and MZ using a specific model of photon. Grangier et al. (1986, 178-9) puts this divergence in the following way: “if we want to use classical ... pictures to interpret these experiments, we must use a particle picture for the first one (‘the photons are not split on a beam splitter’) ... On the contrary, we are compelled to use a wave picture (‘the electromagnetic field is coherently split on a beam splitter’) to interpret the second (interference) experiment”. Bohr made sense of this kind of perplexing situations through the principle of complementarity, according to which a pair of phenomena are “complementary” if they are observed in distinct experiments and the classical model used to explain one phenomenon cannot be simultaneously used to explain the other. Since the observations and the classical explanations of AC and MZ satisfy the stipulations of the complementarity principle, Grangier et al. (1986) and others have concluded that these experiments affirm the wave-particle duality of single-photons.

Duality has also been articulated without the aid of the complementarity principle. A version of this type of claim utilises two distinct observations in MZ to draw wave and particle analogies of single-photons. As discussed previously, when MZ is configured such that constructive interference is observed at $D1_{mz}$, no photon is received at $D2_{mz}$. If the path-difference is gradually altered for constructive interference at $D2_{mz}$, photon-count registered at $D1_{mz}$ correspondingly declines and finally becomes nil (Dimitrova and Weis 2008, 140). This is accounted as single-photons partaking in interference. However, not all aspects of the phenomenon sits well with the “wave-like” interpretation. One such observation is the detection of single-photon at any given time. This is indeed surprising because according to the analysis, single-photons are supposed to be undergoing interference. In other words, even though the photon-count at each detector varies with the change in the path-difference, detectors either receive a single-photon or no photon at all. No “half-photons” are observed (Scarani and Suarez 1998, 719).[4] This characteristic

4. This way of describing the scenario can be found in other places. For instance, Feynman et al. (2010, 1-7), while discussing the double-slit experiment with electrons, mentions how the detectors do not show “half-clicks”.

observation is interpreted as photons behaving “particle-like”. These “wave” and “particle” observations put together bring forth the duality of single-photons. This well-known version of the duality claim is discussed in several places and is often succinctly summarised with the help of Dirac’s statement “each photon interferes with itself” ([1930] 1958, 9).

The above two versions of duality claims appear to have many aspects in common: both are positive claims about photons’ duality and are based on similar experimental observations. In spite of these overlaps, the claims are fundamentally different from one another. While interpreting duality through complementarity, Aspect and Grangier (1987) do not explicitly discuss the alternative version, but they do mention in passing that the conflicting observations in MZ, which provide the ground for both the analogies, are not “the most relevant result for wave particle duality” (ibid., 16). At the same time, much opposed to the first version, there have been several recent arguments — like Ghose and Home (1992) and Dimitrova and Weis (2008) — that demonstrate duality in non-complementary scenarios. Therefore, even though these two versions of claims have usually been contrasted based on their dependency on complementarity principle, the difference in the way duality gets grounded in each of these claims has not been recognised. In the second version, it is just the conflicting analogical observations that goes into the formulation of the duality. The need of these observations belonging to mutually exclusive experiments, which is crucial for the first version, is not a necessary criterion for the second version. Hence, among the two variants, the latter is clearly a more generalised version of duality and the former is comparatively more restricted due to the presence of an additional criterion. Recognising the ground of duality in each of these claims suggests how to carry out the main task of the present chapter — evaluating the duality of photons. In the first version, the relation between complementarity and duality needs to be examined, which will be undertaken in section 5.3. In the second type of claim, the soundness of the analogical claims needs to be scrutinised and this will be done in sections 5.5 and 5.6.

Before proceeding further, it is helpful to recognise the distinct nature of duality found in both these claims by comparing it with the other notions of duality. Here, since Bohr’s notion is the popular one among the two versions of the concerned duality claim, I will compare this with the other connotations. For Bohr, the wave-particle duality arises due to the non-suitability of the classical concepts for describing quantum phenomena. This interpretation differs categorically from the initial notion of duality proposed by Einstein ([1909] 1989, 394), according to which radiation possesses “two structural properties (the undulatory structure and the quantum structure) simultaneously”. The concerned notion of duality can also be differentiated from the one held by Heisenberg and Dirac. Even though it was thought earlier that Heisenberg endorsed Bohr’s views, Camilleri (2006) argues that he later favoured the equivalence — not the complementarity — of “wave” and “particle” interpretations. Dirac, who was in conversation with Heisenberg throughout this period, expressed similar opinion about duality (Bromberg 1977; Kragh 1990, 338n40). Even though this discussion about the diverse notions of duality is not exhaustive,[5] these comparisons bring to the forefront interesting features of the concerned notion of duality. First of all, the subject of this duality claim is not radiation in general, but photons. It is important to note this because in other variants listed above, the commitment to the reality of light-quantum was one of the rationale for proposing duality of light. Contrary to these cases, the concerned duality is about photons that also exhibits "wave-like" behaviour. This significant shift — from being a factor for duality claim to being the very subject of it — has not been noticed at all. The other feature is about the duals — the "wave" and "particle" aspects — that constitute the duality of photons. Since this duality is about a complementarity scenario, the duals are experimental observations. Unlike this case, in Einstein's notion, the duals are physical constituents of electromagnetic field: "singularities" and the "field of force" surrounding them (Einstein [1909] 1989, 394).[6] The duals in Heisenberg's articulation are two equivalent theoretical frameworks about quantum mechanics. With these general observations about this duality, in the subsequent sections, the two versions of this claim will be dealt in detail.

5.3 Critique of Duality Based on Complementarity

After elaborating on the nature of duality about photons, it is time to evaluate these claims. In this section and the following ones, I will analyse them in detail and argue that these wave-particle duality claims are untenable. In the current section, I will explore the duality claim based on the complementarity principle and highlight its problems. With the help of the criticisms already available, I will discuss the difficulty of considering duality as a legitimate complementarity scenario. Also, the problem of inferring duality from complementarity will be shown.

In order to consider the behaviours in AC and MZ as an instance of wave-particle complementarity, it is necessary to clarify that these observations are "incompatible" behaviours of photons in two "mutually exclusive" experiments (Grangier et al. 1986, 179; Aspect and Grangier 1987, 15). The observations can be shown to be *mutually exclusive* based on the detectors' photon-count. In MZ, each detector's probability is decided by the path-difference and this is labelled as the "wave" behaviour of photons. In the same experiment (with no change in the path-difference), if attempt is made to detect which

5. Apart from the above formulations, there are other interpretations of wave-particle duality that, in subtle ways, differ from the ones mentioned. For instance, there are formulations of duality that are contextualised within X-ray scattering experiments during the first two decades of twentieth century (Wheaton 1991). Another important formulation is that of Louis de Broglie. For more about his interpretation and how it influenced other notions of duality, see Darrigol (1986).

6. For more on Einstein's picture of radiation being constituted of energy quantum surrounded by the "ghost field", see Dongen (2007).

path each photon takes, then the interference pattern is lost and the photon-detection across the detectors will become random. Since this observation at the detectors is similar to the one observed in AC, photons are considered to be not interfering any more, but behaving like "particles". Hence, these observations are mutually exclusive as it is not possible to simultaneously observe the interference pattern and know the path taken by photons. These observations also give rise to *incompatible* descriptions of photons. Here, it should be noted that not every description of photons' behaviour in MZ and AC will turn out to be incompatible. For instance, "quantum mechanical descriptions" (one that uses density-matrices or state-vectors) of these experiments are compatible (Aspect and Grangier 1987, 15). Only when classical models of particles and waves are used, the descriptions turn out to be incompatible. The stipulation to use classical models comes from the tenets of the complementarity principle.[7] Even though this prescription might look ad hoc, there are grounds for using these analogical models in the specific phenomena.[8] Thus, these two observations constitute a valid complementarity situation given that they are mutually exclusive and incompatible descriptions. What makes this complementary situation a *duality* phenomenon is the presence of the complementary "wave" and "particle" pictures in them.

Therefore, the claim of wave-particle duality here draws its legitimacy from being a genuine complementarity situation. However, there are serious flaws in this interpretation: duality does not satisfy both the criteria of complementarity; and moreover, Bohr, in his later reformulations of the principle, eschews duality as one of the valid instance of complementarity. In the initial proposal, Bohr held the mutual exclusion and the joint completion criteria to be the essential features of complementary descriptions.[9] *Mutual exclusivity* demands a pair of descriptions to be logically contradictory to one another and *joint completion* indicates the simultaneous need of these descriptions to completely describe the concerned atomic object or phenomenon (Held 1994, 874). From the beginning, Bohr was aware that these two criteria are inconsistent with one another and made several attempts to resolve this. In 1935, he completely reformulates the principle by removing the joint completion criterion and further restricts the scope of the mutual exclusivity criterion. In this revised state, the "mature conception" of complementarity was no more applicable to the wave-particle duality problem. As a result, post 1935, Bohr "tacitly

7. Ghose and Home (1992, 1442) mention Bohr's emphasis to use classical pictures to describe quantum phenomena. However, this suggestion about Bohr's emphasis goes completely against Held's analysis. Held argues that Bohr's "interpretation of quantum mechanics expresses, rather than transcends, the limits of classical concepts" (Held 1994, 879).

8. Aspect and Grangier (1987, 8) and Ghose and Home (1992, 1438-9) provide reasons for using these models. I will examine these analogies extensively in the next section.

9. This representation of complementarity is not distinctly articulated in the Como lecture given by Bohr (1928). As Beller (2001, 148-49) has argued, "the main message of the Como lecture was neither the democratic solution of the wave-particle dilemma, nor the 'wholeness' of the experimental arrangements. . . These are, indeed, Bohr's later elaborations of his thought that often used in a 'backward' fashion to clarify the meaning of his original ideas".

abandons" duality from the discussions concerning this principle (Held 1994, 880-85).

The alteration of the principle and the eventual renouncement of duality as a complementarity scenario points to the struggle of interpreting it within the framework. Regarding this, three problems have been identified. One of the founding assumptions of the complementarity principle has been the mutual exclusivity of two observations, which are otherwise not exclusive in classical scenarios. The complementary pairs like spacetime-causal descriptions and momentum-position values validate this assumption. However, wave-particle descriptions do not uphold this requirement. This kind of pairs is mutually exclusive not only in quantum mechanics, but also in classical theories (ibid., 881). Duality also do not strictly comply with the mutual exclusivity's demand of complementary observations being not part of the same experiment. As Bohr himself notes, it is possible to observe both "wave" and "particle" features in the double-slit experiment (Bokulich 2017, 139). As Held (1994, 881) mentions, "some properties of both pictures are somehow 'blended': the determinate position of each particle that hits the screen certainly is a particle property; the distribution of all impacts exhibits interference, thus is a wave property".[10] Lastly, the joint completion criterion cannot be meaningfully applied in wave-particle duality scenarios. In other situations — like for instance spacetime-causal complementarity — the two observations put together do provide in a way the "complete picture" of the phenomenon involved. However, when this criterion is used in duality scenario, it is not clear in what sense the "wave" and "particle" descriptions jointly complete one another (Bokulich 2017, 139).

The above discussion provides sufficient reasons to doubt the duality claim of single-photons situated in the complementarity principle. Without paying attention to these conceptual difficulties and historical details, several papers — like Grangier et al. (1986) and Aspect and Grangier (1987) — have argued for the duality of photons. These papers have presumed that the establishment of complementarity necessarily implies duality. This premise is problematic as there is no logical relationship between complementarity and duality. Bohr, being interested in providing a framework to consistently bring together certain observations, proposed the complementarity principle. The principle elucidated specific situations that were already observed in quantum mechanics. In this sense, the principle did not imply duality, but rather strengthened a known observation "by rendering it slightly more precise" (Bunge 1968, 276). Because of this, Bunge thinks complementarity should be qualified as a "pseudoprinciple" as "it entails nothing . . . no theorem follows from it". The conceptual distinctness of complementarity and duality can also be seen in

10. The second version of the duality claim formulated solely based on the analogical interpretation of MZ's observations (discussed in section 5.2) is a recent illustration of this failure. Ghose and Home (1992) have also argued against the complementarity principle based on the same ground. In this paper, the authors showed the non-mutual exclusivity of photons' wave and particle behaviours using a two-prism experiment. Even though the authors provide a novel experimental proof to illustrate the weakness of the initial formulation of complementarity, their work is historically uninformed about Bohr's own rejection of the same.

the context of Bohr's revision of the principle. When Bohr reworked the principle and concluded that duality does not fit within the framework, this was merely the denial of duality as a complementary situation and not a rejection of duality as a concept. Therefore, duality and complementarity are different and one concept does not necessarily entail the other.

5.4 Nature and Evaluation of Analogies

The previous section analysed the duality claim that was couched within the complementarity principle. With that approach criticized, the force of the duality about single-photons now solely rests on the strength of the analogies used in the claim. An instance of this type of claim was mentioned in the section 5.2, where duality was constituted by two analogical claims — photons being "wave-like" and "particle-like" — that were articulated in the context of MZ. To mention another instance, Ghose and Home (1992) discuss duality in a novel experimental scenario, where the particle analogy is still based on photon's anti-correlation, but the wave-behaviour is situated in photon's tunnelling phenomenon. As these examples show, duality can be claimed by illustrating the applicability of the respective analogies for describing photons. Thus, the evaluation of the wave-particle duality boils down to critically analysing its constitutive analogies. And this is the task of the present and the subsequent sections.

Analogy, in general, indicates the similarity between various characteristics (like components, properties, relations) of two domains (like things, concepts), which are usually labelled as "source" and "target" (Hesse 1970; Bartha 2016). The formulation of an analogy is usually enabled, and at the same time constrained, by few aspects (Holyoak and Thagard 1995, 15). First, establishing an analogy demands some overlap between the entities involved. Even though this similarity can be established through various means, it needs to be structurally consistent (i.e., be isomorphic) (ibid., 29). Another crucial aspect that shapes the analogy's formulation is the purpose for which it is being evoked. This goal decides not only the suitability of the selected source for the given target, but also the eventual success of the analogy drawn (ibid., 35). With these constraints identified, in the present section, I will discuss the purpose of the concerned analogies and identify few general characteristics about them. After this, I will summarise the strategy adopted to evaluate the structural consistency of these analogies.

Foremost, the "wave-like" and "particle-like" claims about photons found in the concerned papers do not play any significant role in the physically understanding the respective phenomena. Analogies generally have been vital for modelling unobservable theoretical entities[11] and in this regard, wave and particle models were the prominent interpretations

11. For a recent work on the use of analogies in model building, see Bailer-Jones (2009), especially the third chapter.

of radiation up until twentieth century. Even though, the concerned analogies have lineage in the above historical enquiry, they do not have any of the reality bite that their historical predecessors possessed. This is because these analogies do not have any role in the theoretical analysis of the concerned experiments. Quantum optics analyses photons' behaviour across a beam-splitter, which is the central phenomenon in both AC and MZ, through the transmission and the reflection coefficients of the beam-splitter. These coefficients for a specific scenario are largely determined by the principle of energy conservation along with the assumption that photons can be detected only once (Loudon 2001, 216). In the case of photons' interference, quantum mechanics provides an interpretation which is quite unlike the classical understanding through waves. Thus, these analogies do not serve any physical purpose and are incidental to the study of AC and MZ. This status of them is implicit in the concerned papers. None of these discussions naively claim that photons are actually waves or photons. Instead, the assertions are just that some similarities can be drawn between photons and waves/particles in certain scenarios. For instance, as discussed in section 5.3, these "classical" analogies were used in the relevant contexts only to make the observations contradictory to each other in a particular way.[12]

The clarification about the purpose of the concerned analogies brings forth important characteristic about them. The analogies not having any physical relevance and being just claims about similarities implies that the meaning of the claims is not evident. That is, in the concerned contexts, when "wave-like" and "particle-like" are invoked, it is not immediately clear which aspect of photons and of waves/particles are being compared. This is because, neither the claims are strong ones, where mapping of all the essential characteristics of the source onto the target is suggested, nor specific similarities are uniquely determined by the respective theories. With there being numerous similarities between these, it is not definite which one is referred here. Given this, the only way to ascertain the intended similarity is to analyse the context of the invocation and identify the meaning of the analogy.

The clarity about the context sensitivity of these claims directs how to evaluate them. Since the analogies are specific to the concerned scenarios, a general evaluation of these analogies will not be effective. A case in point is the criticism offered by Bunge (1968) about these analogies. For him, the prevailing confusion surrounding duality indicates that "the particle and the wave analogies . . . seem to have reached their breaking point" (ibid., 269). By discussing the use of these analogies in various scenarios — conflict between Schrödinger's and Born's equations, Compton's effect, de Broglie's analysis and complementarity scenarios — he identifies two general weaknesses about them: they are unsuitable for non-classical contexts and they are "mutually inconsistent" (ibid., 270).

12. As Aspect and Grangier (1987, 15) mention, "the problem of incompatible descriptions arises only if we insist on using classical concepts such as waves or particles. But if we stand to the quantum mechanical description, there is a unique description of the light, by the same state vector (or density-matrix) for experiments. . . ".

Even though Bunge correctly identifies that duality is a problem due to analogies, his diagnosis of the situation — why did this problem arise? — is partly problematic and largely not helpful for the present enquiry. First of all, it is not sufficient to claim that the analogies in duality are mutually inconsistent and thus should be avoided. Because the proponents of duality would point that the inconsistency is the central feature of this non-classical concept. Moreover, Bunge offers no argument for the unsuitability of these analogies for quantum mechanics. Here, he does not show the problem of using these analogies in a specific theoretical or experimental scenario. Instead, by stating that "analogical thinking, however fertile as a starter, eventually becomes confusing", Bunge calls for banishing all analogies from science (Bunge 1968, 280).

Bunge's criticism indeed caution us about the concerned analogies. However, a general argument is not enough to reveal the nature of the issue in every instance of duality. That is why, an argument specific to the concerned situation is required and this is what I will do in the subsequent sections. As already mentioned above, the distinct relations between photons and waves/particles drawn here are not evident. In this state, a pragmatic approach is to identify few possible aspects of AC and MZ that might provide the ground for establishing the similarity. As I will show, in AC, two places where similarity between photons and particles can be drawn are the anti-correlation of detectors and "non-splitting" behaviour of photons at beam-splitter. And in MZ, photons undergoing interference phenomenon and observation of fringe-pattern similar to the classical interference pattern are two possible places where analogy between photons and waves can be formulated. Even though there are multiple possibilities, in each of these, there are hurdles in demonstrating the similarity between the source and the target. As I will argue, some of the grounds for similarity — like photons showing anti-correlation and partaking in interference phenomenon — in spite of appearing initially viable, should be avoided. Other similarity-grounds — like the "indivisibility" of photons and "non-splitting" of particles — do not establish an distinct analogy. And in the remaining cases, strong similarity can be established. However, the relations that can be drawn in these are not between the required source and the target. For instance, the context of the "non-splitting" behaviour in AC favours an analogy between single-photons and classical-waves, and not particles. And regarding the fringe-pattern in MZ, it are not single-photons, but an ensemble of photons that are similar to waves. Thus, there seems to be no good reason for holding that single-photons are like "wave-like" and "particle-like".

5.5 Critique of Particle Analogy

As discussed in section 5.2, single-photons being "particle-like" is illustrated both in AC and MZ: in AC, photons exhibiting anti-correlation and in MZ, the non-reception of "half-photons" at the detectors. Even though these claims are different, both of them are characterising the same aspect of photons — their "single particle-like propagation" (see Ghose and Home (1992, 1438)) in the experiments. Since AC is regarded as the definitive illustration of "particle-like" behaviour and a much simpler experiment compared to MZ, I will focus on it for the present enquiry. Before evaluating this analogy, I will first argue for the general unsuitability of the particle analogy for photons in AC through the case study of the HBT controversy. Proceeding further, regarding the analogy between photons and particles based on their "non-splitting" behaviours, I will analyse the similarity using two different interpretations of this notion: the general notion of indivisibility and the operational notion of non-splitting. In both these ways of understanding "non-splitting", I argue that the similarity between photons and particles cannot be distinctly established and thus, the concerned analogy looses its force. Here, I also show that the operational analysis of "non-splitting" in AC suggests that, in contrast to classical-particles, classical-waves is a better candidate for formulating the analogy with photons.

Questioning the candidacy of classical-particles for describing photons' behaviour in AC might seem frivolous given the historical precedence of interpreting photons as particles. However, the unsuitability of the particle analogy in the very context of correlation experiments was realised during late 1950s. This concern came to forefront with the publication of a paper by Robert Hanbury Brown and Richard Twiss in 1956 about an experiment (henceforth *HBT*) they conducted. The authors showed that even when feeble light is used in their newly designed interferometer, positive correlation between intensity fluctuation of the beams in both the arms could be established (Hanbury Brown and Twiss 1956). When the experiment was interpreted using particle model of light, the observation implied that photons should split at the beam-splitter. Since single-photons were still conceived during that time as "distinguishable entities . . . indivisible objects", HBT's conclusion faced severe criticism (Silva and Freire 2013, 456). Edward Purcell, responding to this controversy, argued that the HBT observation makes sense if the intensity fluctuations were interpreted statistically. The observation is merely an outcome of "clumping" (i.e., bunching) of photons (Purcell 1956, 1450). In order to establish this point, Purcell considered three different scenarios of a split-beam experiment (similar to AC), each one with different kinds of incident beam — that of classical-particles, fermions (e.g., electrons) and bosons (e.g., photons). Given that each of these entities exhibit different statistical behaviours (classical, Fermi-Dirac and Bose-Einstein statistics respectively), the "cross-correlation behaviour" (i.e., simultaneous detection at detectors) observed in each of the respective trial would be different. Having made this distinction, Purcell showed that the original fluctuations observed in HBT clearly exhibit bosons' behaviour. The HBT confusion and Purcell's intervention brought forward the drawback of using the classical image of photons. The controversy arose because physicists at that time conceived light as "a stream of discrete indivisible, corpuscular photon" (Silva and Freire 2013, 488). Instead of this "crude model", light has to be strictly considered as a stream of bosons.

Therefore, the analogy drawn between photons and particles in the HBT scenario is not structurally consistent. The lesson learnt from this controversy provides good motivation to abstain from the particle analogy for photons in correlation experiments like HBT and AC. However, the above argument provided in the context of HBT cannot be directly used for criticising the analogy drawn in AC as there are two differences. In HBT, even though the initial confusion was about "splitting" of photons at beam-splitter, the conclusion that photons are different from particles was argued by showing that the correlation observations are not the suitable similarity-grounds for the analogy. Also, the conclusion drawn in this scenario is a general one: photons and particles have essentially different correlation statistics. In contrast to this, the concerned particle analogy claim is articulated specifically in the context of antibunched-light. And more importantly, as will be shown below, the similarity-ground for the concerned analogy is the behaviour of photons at beam-splitter. For this specific scenario, no argument about the weakness of using the particle analogy have been provided. And this is what I will undertake now.

In AC, there are largely two phenomena — photons' movement through B_{ac} and their eventual detection — that can be the possible grounds for claiming that single-photons are "particle-like". Among these, the anti-correlation of detectors should be avoided for comparison. As the HBT controversy emphasised, the correlation statistics of particles and photons, in spite of the superficial similarities, are categorically different. Moreover, the anti-correlation is not a singular observable event which can be isolated and identified as "particle-like". This is inferred based on the non-classical statistical analyses of ensemble averages (Grangier et al. 1986, 175; Loudon 2001, 218). In any case, the particle analogy claim is usually drawn in the context of the other phenomenon at AC: single-photons not getting "split" at B_{ac} (Grangier et al. 1986, 178; Aspect and Grangier 1987, 14). None of the concerned papers, however, have elaborated on how this similarity works and have simply presumed its soundness. Concern about this claim arises because the jump from photons' "non-splitting" to being "particle-like" is unclear. First of all, what are the grounds for interpreting the photons' behaviour at B_{ac} as a "non-splitting" phenomenon? And even if this interpretation is considered as the starting point for drawing the analogy, why particles should be preferred as the source for the analogy? Here, in what sense photons not splitting is parallel to that of particles? As there are plural interpretations of "non-splitting", which specific meanings are being compared? Answering these questions is crucial not only for demonstrating the possibility of this analogy but also for assessing its strength. In order to do so, I will check the possibility of providing similar but yet meaningful interpretations of "non-splitting" for both these entities. Two interpretations will be considered: the general notion of indivisibility and the operational notion of "non-splitting". In both these cases, I will argue that similarity between single-photons and particles cannot be established. Hence, the concerned analogy does not even take off.

Since classical-particles were invoked as the analogy's source, it might be suggested that "non-splitting" in AC represents the fundamental aspect of *indivisibility*. If this interpretation is used, the concerned analogy would be about photons and particles being similar as they exhibit some kind of indivisibility. Even though this interpretation adopts a much more definite notion of "non-splitting", the structural similarity of this analogy is still not clear for two reasons. Indivisibility of particles, like "non-splitting", has multiple meanings. There are general notions, like mereological (not further made up of parts) and mechanical (inability to divide using any force) indivisibilities, and more specific ones, like material and spatial indivisibilities. Among these, it is uncertain which one is being used here. On the photon side, the concern is much more basic: it is still not sure whether indivisibility is applicable for these discrete yet variable quantities of energy and if possible, how to define it. Given this status, it is unwarranted to read the "non-splitting" observation as a claim about single-photons' indivisibility. To do so would be as erroneous as inferring the indivisibility of particles merely based on their non-splitting behaviour in a double-slit experiment just because other entities like regular-light waves split. Probably for these reasons, none of the concerned papers make this strong interpretation about photons' behaviour or mention "indivisibility" while drawing the analogy. Therefore, the claimed analogy cannot be established by interpreting "non-splitting" as indivisibility in the concerned scenarios.

Alternative to the stronger interpretation of "non-splitting", a much more cautious and straightforward approach is to understand it contextually. This *operational* notion of "splitting", or equivalently "non-splitting", is situated entirely in a specific phenomenon that provides a unique process through which the entity "splits" and thus defines what constitutes as "splitting" in this scenario. In this view, there is no general notion of splitting for an entity as multiple phenomena of "splitting" can be provided. For instance, splitting of regular-light in a double-slit experiment is a different notion compared to the one involved at a beam-splitter. Here, the difference is not only with the experimental context, but also with the mechanism of splitting. Similarly, in the context of photons, if AC is held to bring out their "non-splitting" behaviour, the parametric down-conversion process (of single-photons generation) can be considered as a "splitting" process where a high-frequency photon "*splits*... into two lower-frequency, signal and idler photons" (Lounis and Orrit 2005, 1159, emphasis added). The plurality of definitions instructs that while discussing the "splitting" of an entity, it is necessary to highlight which type of splitting is being referred.

So, unlike the general interpretation that demanded both particles and photons to be indivisible, the operational view understands "non-splitting" specifically for the given scenarios. Establishing the analogy using this notion requires first defining the respective notions of "non-splitting" and then showing the structural similarity between them. In this approach, the concerned particle analogy faces two problems, one concerning the definition of particles' "non-splitting" and the other regarding the preferred source of analogy. Regarding the first hurdle, none of the concerned papers have taken effort in specifying how "non-splitting" for particles should be defined here. Without contextualising this notion in a scenario, "non-splitting" for particles is undefined and with this, the analogy cannot be actualised nor can be evaluated for its strength. This concern pertains to the lack of details in the present formulation and it might be responded that this can be remedied by furnishing the suitable characteristics of non-splitting. However, this approach of first picking the source of the analogy and later rendering it suitable seems ad hoc. Because, the selection of the source — an entity with a specific kind of non-splitting — should be based on what the scenario of the target — the "non-splitting" of photons in AC — demands. Not doing so and using an archetype source is like presuming a viscous-fluid, which does not "split" at a non-permeable filter, to be "particle-like" without analysing further the notion of "non-split" in this scenario. None of the concerned papers have analysed "non-splitting" of single-photons relevant to AC and without any justification, have preferred classical-particles as the source for the analogy. This is a serious lapse because, as I will show now, the very analysis of "non-splitting" at AC suggests another suitable source for an analogy.

What does "non-splitting" of single-photons in AC mean? This description does not literally capture photons' behaviour at B_{ac}. Instead, it describes the photons' transmission through the negation of another behaviour — as "splitting" *not* happening. The preference to use this phrase can be understood by situating AC in the history of QFT. CFT held that correlations will be observed in AC since feeble regular-light — being modelled as classical-waves — splits at B_{ac} and triggers both the detectors. QFT, on the other hand, considered radiation to be quantised and suggested the possibility of anti-correlation. With these theories predicting different observations, anti-correlation phenomenon became a key experiment that could evaluate them. An experiment along this line was carried out by John Clauser (1974). In this landmark paper, to show that classical and semi-classical theories are wrong about the quantisation of field's energy, Clauser designed an experiment similar to AC to demonstrate that "a photon is not *split* in two by a beam splitter" (ibid., 855, emphasis added). The underlying presumption here was that "a particle must be either transmitted or reflected. Both may be done simultaneously only by a wave" (ibid., 854). The successful observation of anti-correlation demonstrated that photons do not "split" at B_{ac}. As the brief overview contextualises, this notion of "non-splitting" is situated in a scenario where regular-light "splits" at beam-splitters, i.e. comes out through both the sides of B_{ac}. Here, "split" is primarily a feature of regular-light that is uniquely defined in the context of AC. In the same scenario, since antibunched-light does not exhibit correlation, single-photons coming out through either sides of B_{ac} were interpreted as "non-splitting". Thus, this qualification, as the negative prefix indicates, actually signifies photons' deviance from regular-light in the concerned context. The comparative nature of the qualification, therefore, points that the very description of photons as "non-splitting" is an analogical way of capturing their deviance from regular-light.[13] As regular-light is modelled on classical-waves, "non-splitting" of photons can also be expressed as an analogical relation between single-photons and waves.[14]

The above discussion clarifies that if there is a need to analogically describe the behaviour of photons at beam-splitters and if the criterion for similarity (or dissimilarity) is considered to be "non-splitting", then regular-light or classical-waves are the suitable sources. Since the corresponding description — "single-photons are *non-wave-like*" — is more suitable, this suggestion is a *counteranalogy* to the proposed "particle-like" analogy found in the concerned papers. As Shelley (2002, 487) discusses, counteranalogy is "an alternative hypothesis" that is "more acceptable" in the given context compared to the proposed analogy. Indeed, there were few historical motivations for using the particle analogy in the concerned context. Since particles have been traditionally considered as the antithesis of waves, it is understandable that particles were preferred when photons were found to be different than waves. Apart from this factor, as mentioned, there was classical tendency to interpret photons as particles. Given these, it is not surprising to find particle analogy being invoked for photons here. Nevertheless, what I have argued for in this section is also not unfamiliar. The present conclusion is similar to other recent arguments that highlight the prematurity of considering photons to be particle-like. In this sense, what I have carried out is a contribution to the series of corrections undertaken to gradually undo the mistakes regarding the conceptualisation of photons.

5.6 Critique of Wave Analogy

The claim that single-photons are "wave-like" mainly arises due to their behaviour observed in MZ. The interference observed here with regular-light has traditionally been understood through the classical-wave model of light. Given that similar fringe-pattern is observed even when antibunched-light is used, single-photons have been considered to behave like waves. Among the concerned papers, there are largely two ways through which this analogical claim has been established. In this section, I will analyse both these approaches. The first approach, I argue, adopts an inappropriate starting point for formulating analogies. In contrast to this, the second approach does identify a possible similarity between photons and waves. However, in this ground, the concerned wave-like claim turns out to be a false

13. Given that it is not similarity but difference that is being accounted here, the relation drawn between photons and regular-light can be considered as an instance of *negative* analogy, to use a terminology suggested by Hesse (1970, 8).

14. Similar kind of analysis can be carried out for the classical double-slit experiment, where waves and particles are distinguished based on the fundamental characteristic of waves to "split" at the slits. Here too, particles are considered to be "non-splitting" only with reference to waves. Because of this derivative nature, this demonstration cannot be utilised as a general illustration of particles' "non-splitting".

analogy.

For the evaluation of the concerned analogy, the primary requirement is to understand how similarity between photons and waves can be demonstrated in the given context of interference. This relation has to be drawn carefully as photons and waves belong to rival theories of radiation. Since these theories provide distinct explanations of the same set of observations, the respective entities partaking in alternative descriptions of an observation should not be interpreted as being analogous. Emphasising this caution is necessary as the duality claim based on the complementarity principle seems to use this strategy to show similarity between photons and waves. As can be seen in Grangier et al. (1986, 178-79) and Aspect and Grangier (1987, 14), photons' behaviour at MZ is considered to be "wave-like" because the alternative description provided by the classical theory of light seems to equally explain the observation.[15] Of course, this way of comparing photons and waves is legitimised in complementarity by stipulating the usage of classical models for interpreting a given observation as an essential requirement. However, outside the complementarity framework, drawing similarity like this leads to problematic claims. This approach invariably makes every pair of competing entities and respective theories "analogous" to one another. For instance, photons can be considered "wave-like" because waves (in semi-classical theory) too can explain the photoelectric effect. And with respect to the present case, this approach suggests single-photons are analogous to waves since they both exhibit interference. This is a misleading claim because there is no obvious overlap amidst the alternative explanations. Classically, interference (henceforth I_c) with regular-light is interpreted as two waves spatio-temporally superposing at $B2_{mz}$ and producing a fringe-pattern due to the path-difference between the arms. In quantum theory, interference (henceforth I_q) is an outcome of single-photons being in quantum superposition states. Since a photon has two possible ways to reach the detector in MZ, its quantum state is described through the superposition of two states, one for each path it can take. This is how the path-difference plays a role in deciding the probability of each photon's detection. With this, the pattern observed at the detectors is interpreted as a collective outcome of single-photons exhibiting their quantum behaviour individually. As this discussion shows, I_c and I_q disagree at every level: not only about the meaning of "interference" but also about the phenomenon that unfolds in MZ. Thus, as these are just parallel explanations, it is inappropriate to consider the corresponding entities as analogous. Also, there is another reason to avoid considering single-photons here to be "wave-like". As in the context of AC, in MZ too single-photons behave "non-wave-like": they are received always in either of the detectors; no "half-photons" are observed. Given this, it would be contradictory to say that single-photons behave like "wave-like" too in

15. For instance, Aspect and Grangier (1987, 14) mention, MZ "can only be understood in the framework of a wave theory ('the electromagnetic field is coherently split on the first beam splitter, and recombined on the second, and this recombination depends on the path difference')".

MZ.

The above discussion instructs how not to draw an analogy between photons and waves in the concerned context. There have been other similar analogy claims that do not make this mistake. An instance of this is found in Dimitrova and Weis (2008, 140), where photons are considered to be wave-like as they exhibit "the familiar two-wave interference fringes" in MZ. In this type of claims, instead of naively claiming that photons and waves exhibit "interference", the fringe-pattern developed in the case of antibunched-light is being compared with the patterns traditionally developed using regular-light. This comparison, however, is not straightforward since these patterns not only have different appearance, but they are produced in experimental setups that have different sources and detection processes. Given that the patterns are distinct, the similarity between them needs to be established. As I_c uses regular-light, fringe-patterns on the screens at either ends of $B2_{mz}$ are observed for a specific path-difference. In I_q, which uses antibunched-light, the pattern is generated over the entire duration of the experiment by plotting the detection-count of a detector against the path-difference (Grangier et al. 1986, 178). A point on the pattern, which is produced by the whole ensemble of photons, represents only the number of single-photons received over the counting-time for a certain path-difference. Unlike the fringes in I_c which are observed for a path-difference, the fringes at I_q are gradually produced — the "apparition of the interferences 'photon by photon'" — when the path-difference is varied throughout the duration of the experiment (Aspect and Grangier 1987, 16).[16] Thus, the patterns in I_c and I_q have different process of formation. More importantly, the characteristic features of each pattern (like for instance a particular point on the pattern) have different meanings in the given experimental context. In spite of these differences, the fringe-pattern produced at I_q can be considered similar to the one seen in I_c as this captures — in its unique way — one of the features of interference phenomenon: path-dependent variation of specific quantity (i.e., photon-detection).

From the above discussion, it follows that an ensemble of single-photons is analogous to waves. However this claim is unlike the concerned wave analogy found in the duality claim, i.e., "single-photons are wave-like". The possible claim and the concerned one differ about the target of the analogy. This implies that, in the context of similarity between photons' and waves' fringe-patterns, the concerned claim turns out to be a *false* analogy (Shelley 2002, 489). An analogy is "false" if any of its condition — structural consistency, similarity between target and source, and its purpose — turns out to be erroneous. A structurally consistent similarity can only be established between an ensemble of single-photons and waves, and not between single-photons and waves.

16. This observation is unique to interference phenomenon of single-photons. For instance, Pipkin (1979, 294), whom Grangier et al. (1986) point to for a summary of past interference experiments, describes the observation at I_q as "the interference pattern formed by *integration of many events* in which there is only one photon in the apparatus at a time . . . " (emphasis added). Other interference phenomena, like that of two-photons, do not exhibit this characteristic fringe-pattern (see Paul (1986)).

5.7 Conclusion

In the last two sections, I have argued that the concerned analogical claims about single-photons — being "particle-like" in AC and "wave-like" in MZ — turn out to be either incomplete or wrong. Since no strong justification is found, the wave-particle duality claim — "single-photons are wave-like and particle-like" — becomes weak in the given context. In contrast to the concerned analogies, I have shown that in AC, each photon behaves "non-wave-like" and in MZ, it is not single-photon but an ensemble which behaves "wave-like". Both of these possible analogies seem to be appropriate even outside the respective scenarios in which they were illustrated. Single-photons behave "non-wave-like" not only in AC, but also in MZ. And, the conclusion about photons from MZ seems to be applicable at AC as well: since an ensemble of photons in AC recreates the classical behaviour of regular-light "splitting" and exciting both the detectors equally, the ensemble here too shows the "wave-like" characteristics. With the suitability of the possible analogies for both AC and MZ, the behaviour of photons can be summarised in the following manner: even though single-photons are *non-wave-like*, an ensemble of them is still *wave-like*. This description, unlike the duality claim, is consistent among experiments, non-paradoxical and most importantly, captures analogically how photon differs from regular-light.

In this chapter, I have shown how wave and particle analogies were used inappropriately. This kind of casual usage of analogies has reduced AC and MZ to demonstrative experiments that illustrate a caricature version of wave-particle duality. Given this, the arguments presented in this chapter would have served their purpose if AC and MZ are disregarded as "duality" experiments. There are precedences where experiments, once considered to illustrate duality, are no more discussed in that light. To mention one, consider the HBT debate discussed before. In 1957, as a response to the ongoing controversy, Hanbury Brown and Twiss argued that the conflicting interpretations of the experiment should be understood as the complementarity descriptions of the same experiment (Hanbury Brown and Twiss 1957). Given the nature of the debate, it is not surprising that the conflict was subsumed under the complementarity principle and was identified as an instance of wave-particle duality. However, the timely dissolution of the controversy obviated any further labelling of HBT as a duality situation and as a result, none of the contemporary discussions on duality cite HBT as a valid demonstration. My arguments in this chapter should be considered as a similar attempt about AC and MZ.

Chapter 6

Controversy About Interference of Photons

In the previous chapter, I analysed the wave-particle duality claim about single photons and argued how inappropriate analogies were invoked in the respective contexts. This study brought forward one of the many unscrutinised presumptions concerning photons. In the present chapter, I want to closely analyse another aspect pertaining to photons: interference. I will show the ambiguity surrounding interference by illustrating the controversy surrounding the phenomenon. Through a detailed historical study of the three dominant theoretical interpretations of interference, I will establish the controversy and I finally conclude by identifying the important sources of this controversy.

6.1 Confusion about Interference

Interference has been one of the central phenomenon that has guided the understanding of light right from its discovery. During the nineteenth century, this phenomenon provided the decisive ground for the establishment of the wave-interpretation of light. Even in the context of quantum mechanics, which superseded the classical theories, this phenomenon and the related experiments have played fundamental role in the theoretical clarification. Both the classical interpretation (I_c) and the quantum theoretical interpretation (I_q) of interference have already been discussed in section 5.6. While analysing the "wave-like" claim about single-photons, I showed how I_c and I_q are two completely different accounts of what happens in an interferometer. Given that there are no overlaps between I_c and I_q, it needs to be asked whether we should consider them as "different interpretations of *interference*"? That is, even though both are explanations housed in different theories, why is that we should consider them to be different explanations of the same phenomenon *interference*?

Without paying attention to this aspect, physicists have continued to refer the phenomenon at MZ, even when interpreted through quantum mechanics and QED, as "interference". For instance, Grangier et al. (1986) labels MZ as "single photon interference experiments". They refer to Pipkin (1979) who provides a summary of all important interference experiments with photons, from 1909 to 1969. Here, Pipkin illustrates how most of these historical experiments positively show the formation of "interference pattern" and demonstrates that quantum mechanics successfully accounts for the "photon interference effects" (ibid., 301). Most of these discussions base the interpretation of the phenomenon on the analysis of Paul Dirac, who in his 1930's book argues that each photon passing through an interferometer "interferes" with itself. Going back to Dirac for an interpretation of photon's behaviour is obvious given that Dirac, along with Pascal Jordan, in the late 1920s, laid the foundation of quantum theory of radiation. Through quantum mechanical analysis, which I will discuss in detail subsequently, Dirac concluded that "each photon then interferes only with itself. Interference between two different photons never occurs" (Dirac [1930] 1958, 9).

The continued usage of "interference" across theory-changes has serious repercussions. Post 1960s, with the advent of quantum optics, one confusion pertaining to this usage surfaced. Due to the development of techniques to produce independent coherent sources of light, it was possible to show that independent attenuated light beams exhibit "interference". In these experiments, it is not through fringe pattern observation, but specific correlation statistics that indicated the interference of two independent photons. This observation suggested that the second part of Dirac's dictum about interference between two photons might be wrong. For instance, in a paper reviewing various experiments on interference of photons, Paul (1986, 209) mentions at the beginning itself that the second part of Dirac's assertion "cannot be held upheld as a general rule forbidding independent photons ... to interfere". In a more recent work, Hentschel (2018, 149-150) mentions the "frequently quoted dictum was thus quite definitely falsified". In contrast to this position, there are physicists who think otherwise. To quote an example, with respect to a recent experiment about independent photons interference, Wallace (1994, 950) mentions that it is wrong to infer from this experiment that "two different photons can interfere, contrary to Dirac's dictum".

The above discussion indicates a confusion regarding the understanding of interference in the context of photons. This confusion has not been resolved because the actual disagreement between these groups has not been properly identified and given due concern. The above parties appear to be disagreeing on the validity of Dirac's statements. However, as I will show, the actual disagreement is about the definition of "interference" in the context of photons. It is the ambiguity and disagreement about what qualifies as interference phenomenon that is giving rise to this confusion. For some, "interference" stands for the observation of fringe pattern; and for others, certain correlation statistics indicate interference. The main source of this confusion is the blind use of "interference" without rethinking its meaning for photons. Classically, “interference” stood for a collection of interrelated optical characteristics: a process where two waves “interfere”, the possibility of observation of fringe pattern and as a marker of coherence of these waves. With quantum revolution in optics, even though the nature of radiation was revised in terms of photons, the interference phenomenon was not reinterpreted. It is this lapse which is causing these various interpretative problems.

The present confusion about a phenomenon, which was first formulated in the early nineteenth century, is not surprising because the physics of radiation and matter went through a paradigm shift. With this theoretical restructuring, however, it is not that the older characteristics about interference have to be completely shelved. What is required is an analysis of considering what all previously understood aspects of the phenomenon still holds good in the context of photon interpretation of radiation. The following comment of Joan Bromberg captures this point clearly: “Twentieth-century physics is not simply a tale of the invention of quantum mechanics and its progressive conquest of the fields of classical physics. Rather it tells us of the continued viability of both theories, and their productive interplay” (2016, 245). Contextualising the confusion regarding interference in the paradigm shift is also illuminating for another reason. The transition from classical to quantum schema was a shift where all the inter-related concepts in classical physics would have undergone revisions. This suggests that interference would not be the only one and several other related concepts also would have undergone change. History of optics confirms this hypothesis. A good example to mention here is the ambiguity surrounding photon. The confusion regarding this notion has already been discussed in the previous chapters.[1]

The concept of *optical coherence*, which is closely related to the notion of interference, is another example. One source of confusion regarding this concept during the first half of twentieth century was due to different approaches and attitudes towards optics. Given optics was of interest to both engineers and physicists, coherence had multiple meanings arising from both theoretical as well as operational viewpoints. Bromberg (2016, 247) summarises this confusion in the following manner:

> In 1955, A. Theodore Forrester [mentions that] “Certain widespread misconceptions concerning the nature of light ... seem to have their origin in a rather loose usage of the term coherence” ... Robert H. Dicke [in 1959] held “... there has been considerable misunderstanding of coherence concepts in the past ... And Israel R. Senitzky would write in a paper published in 1962, “[An] unsatisfactory situation exists with respect to the concept of coherence ... because of the various different meanings attached to the word ‘coherent’”.

This confusion regarding the notion of coherence, however, was not only due to the

1. Specifically, see section 2.1.6 and section 2.2.

disciplinary clashes and technical advancement; this was also fuelled by the quantum revolution in optics. There were attempts, especially by Emil Wolf and Leonard Mandel, to clarify the notion of coherence from the classical electromagnetism. Contrast to this, Roy Glauber proposed a completely different notion of coherence from quantum electrodynamics (QED). However, the debate between classical/semi-classical electromagnetism and QED, especially regarding the concept of coherence, reached a state of resolution with QED's notion being suitable not only for regular light, but also for non-classical lights (like laser and antibunched light beams).

The above case studies show how transition from classical to quantum framework results in non-clarity about concepts that get carried over. In this sense, confusion regarding interference is understandable. However, unlike the ambiguity about concepts like coherence and photons, the non-clarity about interference has not yet been recognised. Given this, my attempt in this paper is two fold — to highlight the confusion regarding interference and identify the reasons for this confusion. To begin with, I will discuss in detail three main interpretations of interference: the classical analysis in section 6.2, Dirac's interpretation in section 6.3 and modern quantum optics interpretation in section 6.4. After explaining how interference has been largely understood in the above three historical stages, in section 6.5, I will present the controversy surrounding this phenomenon. With this established, in section 6.6, I will identify two important sources of this controversy.

6.2 Superposition and Interference in Classical Optics

As the above discussion suggests, the paradigm shift from classical understanding to quantum mechanics was one of the main reasons that gave rise to the ambiguity about interference. However, from this it should not be concluded that the concept of interference was clearly defined in classical optics and it became problematic only during modern physics. As the history of classical optics shows, Young's formulation itself was ambiguous and this non-clarity affected the immediate reception of his theory. Nahum Kipnis, in his historical analysis of optical interference, mentions some historians expressing this concern (1991, 18). For instance, Kenneth Latchford, while analysing the poor reception of Young's proposal of acoustical interference, identifies "the singularly unfortunate choice of word used by Thomas Young to describe the mechanism of destructive coalescence; namely, interference" as one of the main reasons. He mentions that "in acoustics, the word 'destruction' to explain the occurrence of certain phenomena, such as missing harmonics or beats, was fairly understandable; at least it could be observed to happen. Interference, on the other hand, especially when used in the context of waves, contradicted one of the most characteristically observed facts; namely, the complete interdependence of intersecting wave systems. If such an idea was difficult enough to conceive in acoustics, it was well-nigh impossible in the case of light, even admitting that it was a wave-disturbance" (1975, 73).

The above comment of Latchford is representative of other historical studies that argue that it is due to this non-clear writing and use of words, Young's theory of interference and the larger wave theory of light was not accepted soon after its proposal, and this happened only a decade later when Augustin-Jean Fresnel proposed a clearer version of the same theory. Even though I will clarify the point raised by Latchford subsequently, it is evident from the comment that one of the central difficulty in accepting Young's proposal had to do with the words he used. Since Young used "interference" in the context of two waves intersecting, he did not clarify how "interference" is different from "interaction" of waves. As Kipnis discusses, this non-clarity was not only problematic during Young's time, but continued later as well. The subsequent historians and physicists did not take effort in distinguishing "interference" from other closely related concepts like "superposition". To illustrate this, Kipnis mentions how historians have variedly related these concepts: for John Worrall, interference was equivalent to superposition of waves and Latchford "reserves 'interference' for Young's optics and 'superposition' for his acoustics" (1991, 18).

Therefore, in classical wave theory of light, these concerned concepts are closely related: *interference* phenomenon is an outcome of the *superposition* of two or more *interacting* waves. To understand the overlap among these concepts, it is important to closely analyse Young's notion of interference. According to Kipnis, "the principle of superposition of waves and the principle of interference were unknown before Young" (ibid., 26). Here, he is critiquing the conjecture drawn by many historians that Young merely generalized a principle that was already known prior to him. For instance, John Herschel in 1830 had argued that the principle of interference is merely attributed to Young because "in science, he who generalises, invents" (ibid., 25). Bernard Cohen had claimed that Newton was the first to use the analysis of interference while explaining the tides in the port of Tongkin and Batsha (1940). However, Newton did not generalise this principle from tides to colour hues and this happened only with Young (Mollon 2002, 807). Reacting to these views and other criticisms about Young's analysis, Kipnis sets out to show the novelty of the principle of interference that Young discovered. The important task here is to make explicit the actual content of the principle of interference since the historians and physicists invoke "interference" without actually clarifying what they mean by it (Kipnis 1991, 75). With this agenda, Kipnis undertakes the task of differentiating the various principles of superposition and how with a new principle Young was able to discover the interference of light. In this historical analysis, what is of importance for the present enquiry is the conceptual distinction that Kipnis draws among interaction, superposition and interference of waves. Therefore, I will not be considering the historical methodology and the various arguments that Kipnis provides for his conclusions. Instead, in the backdrop of this brief historical progression of Young's discovery, I will highlight how these concepts can be distinguished in the classical framework.

Young formulated the principle of interference initially in the context of sound waves in 1799 (Kipnis 1991, 46). The important guiding principle of natural philosophers, during and prior to this period, was the independence of sound waves and water waves (ibid., 37). For instance, two water waves pass one another without affecting each other and two people can still communicate across a room even when these sound waves have to criss cross other sounds present in the room. The acousticians were thus primarily concerned with explaining the independence of sound waves. Kipnis discusses how Lagrange and Bernoulli, prior to Young, came up with the mathematical principles to show that "intersection of pulses changes neither their velocities of vibrations nor the speed of sound" (ibid., 40). In contrast to these acousticians, Young chose to analyse the intermediary process of interaction of waves. The validity of the independence of the intersecting waves was based on the observation that "two waves remained the same *after* their intersection as they were *before* it" (ibid., 46, emphasis in the original). However, when the waves are passing one another, it is observed that there is variation of the cumulation intensities. Young showed that waves do *interact* and at the same time remain independent during the intersection. In order to account for both interaction and independence of waves, Young formulated a novel principle of superposition. The earlier acousticians had understood the phenomenon using the *principle of superposition of motions* of the medium's particles. Young, on the other hand, proposed the *principle of superposition of waves* (ibid., 18). With this mathematical rule, Young recognized interference as this "observable physical phenomenon" where the "perceived intensity of sound, light, or water waves experiences considerable periodical changes" (ibid., 21). Thus, by shifting the focus of analysis to the interaction process, Young was able to discover the temporal interference of sound (ibid., 46).

Articulating interference in the context of light took some more time, since Young faced few challenges. Optical interference required first clarifying the nature of light waves. Initially, Young conceived light based on the analogy of sound waves. However, since considering interference as "coincidence of pulses" did not help him to get the required explanation, he reinterpreted the same in terms of "coalescence of undulations" (Darrigol 2012, 174). This facilitated Young to articulate optical interference as "alternation of positive and negative displacements" (ibid., 181). Therefore,by modelling light using water waves, Young, in 1800, found a better way to account the optical interference phenomenon (Kipnis 1991, 80-81). In 1801, he generalised the interference across acoustic and optical phenomenon by formulating the "law of interference", which Kipnis prefers to call as *principle of interference* (1991, 86). This principle, as formulated by Young, becomes the central component of a "quantitative theory" that attempts to explain the interference phenomenon using the principle of superposition waves (ibid., 22). Thus, the observed periodic variation in the intensity is accounted through the superposition of two components waves. The theory of interference, as Young proposed, contains several aspects: the hypothesis of interference, the rule for locating maxima or minima, the conditions for coherence, the periodicity of intersecting waves, etc. (Kipnis 1991, 22).

With the brief overview of the historical context of Young's discovery, it is possible to state clearly Young's theory of interference. In order to do so, consider the classical double slit experiment to demonstrate the interference of light. The two waves emerging from the slits *intersect* with one another and the *interaction* of their amplitudes at a specific distance from the slits can be evaluated using the *principle of superposition of waves*. The superposition principle dictates that periodic (spatial or temporal) variation of intensity will be observed. It is this observation of the fringes that is termed as *interference*. For observing this physical phenomenon, as the *principle of interference* specifies, certain conditions have to be met, like for instance, the waves should have the same frequency, should originate from the same source and the path difference between them should not be large (ibid., 125). As can be seen, in this description of the phenomenon, every term has a specific meaning. However, this clarity about the terms was not available during Young's time. Kipnis even argues that Young himself was not clear about these concepts by illustrating Young's "vacillation" about the novelty of his "acoustical discoveries". In 1798, he felt the interpretation to be new and in 1802 he states "I was not aware that ... there was so much novelty in the mode of obtaining them, as to deserve the name of a theory or an invention" (ibid., 31). Young changes his position in 1807 and states that the superposition of waves is nothing but "general laws of composition of motions ..." (ibid., 31-32).

6.3 Dirac's Interpretation of Interference

Similar to the above analysis of interference in the classical context, the notions interference and superposition in the context of photons have to be understood. In order to attempt that, the initial task would be to analyse Dirac's reinterpretation of interference phenomenon, which is found in his 1930 book. Here, Dirac begins by providing an account of failure of classical physics by illustrating various non-classical findings, one among which is a phenomenon pertaining to light. Dirac mentions how light cannot be described by wave theory alone and there are some phenomena that suggest light to be composed of "small particles" called *photons*. Each of these have "definite energy and momentum, depending on the frequency of the light, and appear to have just as real an existence as electrons, or any other particles known in physics. A fraction of a photon is never observed" (Dirac [1930] 1958, 2). This "anomalous behaviour" (of having dual characteristics), Dirac clarifies, is not peculiar to light alone and even material particles have wave properties. Thus, the notion of photon in Dirac's book is situated largely in the context of duality, according to which radiation and matter have both wave and particle characteristics. The particle interpretation of light arises largely due to the "the breakdown of classical mechanics — not merely an inaccuracy in its laws of motion, but *an inadequacy of its concepts to supply* *us with a description of atomic events*" (Dirac [1930] 1958, 3, emphasis in the original).

The way Dirac relates the dual features of light is clearly brought out in the discussion of the *principle of superposition of states*, which according to Dirac, is one of the "new set of accurate laws of nature" that grounds quantum mechanics (ibid., 4). Dirac illustrates this principle through the discussion of two phenomena of light: polarisation and interference of photons. Given the present concern of the chapter, I will focus only on interference phenomenon. As already mentioned, Dirac's intention throughout the discussion is to show that the "reconciliation of the wave and corpuscular properties of light" is possible in quantum mechanics through the principle of superposition of states. To demonstrate this, he considers the scenario of single-photons going through the interferometer. Here, the central question to explain is the behaviour of photon which, unlike waves that can split and traverse across the two arms to interfere, cannot go through both the arms. It is in this kind of situations, Dirac suggests, that the principle of superposition of states provides a way forward. According to this principle, photon's state entering the interferometer should be considered as the superposition of two states: one corresponding to photon's state in one arm, and the other capturing its state in the second arm. Each of these states corresponds to the wave-function of classical wave traversing through that respective arm (ibid., 8). Here, "superposition state" indicates that photon's overall state is a probabilistic combination of the two states such that "the photon is partly in one beam and partly in the other" (ibid., 8). And it is the outcome of this superposition of states that leads to what is usually understood as "interference effects".

Through the above analysis, Dirac shows how fringes observed in the standard interferometer experiment can be accounted by the behaviour of single-photons.[2] Dirac, therefore, provides an account of the physical phenomenon happening in the interferometer at the level of single-photons — how the quantum state of a photon passing through the interferometer decides the observational outcome. This understanding explains not only the phenomenon at single-photons level, but also the final ensemble outcome. In this context, it is important to clarify the meaning of "superposition" and "interference". For Dirac, the principle of superposition is a mathematical procedure to express the state of quantum system, very much like the "procedure of resolving a wave into Fourier components" (ibid., 12). According to this principle, quantum state of the system expresses the probabilistic "intermediateness" of being in each of the possible states, quite unlike the classical notion of superposition where the actual value of the state is the intermediate of the possible states' values (ibid., 13). This superposed state, thus, indicates that there exists a "peculiar relationship" among the various possible states of the system. This relationship that the principle explicates cannot be understood through any of the classical

2. This particle reinterpretation of a phenomenon, which was understood classically through waves, should not be thought of as Dirac concluding that radiation is physically constituted by photons. As already discussed in section 4.4, Dirac argues for the equivalence of both the "wave" and "particle" interpretations of radiation.

"pictures" (Dirac [1930] 1958, 10).

Dirac, thus, completely situates interference within the gamut of the principle of superposition of states. However, what does "interference" here stand for? Some parts of the discussion in the chapter suggest that Dirac is referring to the phenomenon at MZ, whatever it might be, as "interference". This phenomenon, which was understood within the classical framework, was completely reinterpreted by Dirac. Irrespective of the differences in these interpretations, given that this kind of phenomenon was traditionally called as "interference", Dirac prefers to refer to it in the same name.[3] Moreover, even though classical optics and quantum mechanics provides different explanations, it is the same "interference effect" — the path dependency of fringe pattern observed — that is being explained.[4] Thus, for Dirac, "interference" seems to be a phenomenon which exhibits certain observations. This interpretation of the phenomenon is quite unlike the classical scenario where the effects are due to the local spatio-temporal interaction of two actual waves. In this sense, interference is no more a phenomenon that involves two entities *interacting*, but about a single entity being in quantum superposition state. Here, Dirac does consider the other alternative interpretation where the wave-function of photons indicate the probable number of photons in either of the arms. In this interpretation, interference is similar to the classical phenomenon since it involves two components — two photons, each coming from either arms — interacting. However, Dirac rejects this interpretation since this does not respect the principle of conservation of energy: "if the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons" (ibid., 9). Dirac concludes that interference should not be considered as a phenomenon that is an outcome of two photons interacting — cancelling and adding up — since it is not clearly how to account for energy conservation here.

The conclusion of the above analysis is captured succinctly by Dirac in the following way — "each photon then interferes only with itself. Interference between two different photons never occurs" (ibid., 9). However, this often quoted dictum of Dirac, surprisingly, contains a usage of interference that is different from the above discussed notion (as a phenomenon having specific observational features). In the dictum, the second statement suggests that the phenomenon at MZ does not involve two photons *interacting* with one another. This connotation of interference — as interaction between two photons — is

3. At the beginning of the discussion itself, Dirac mentions "Let us take a definite experiment demonstrating *interference*" (Dirac [1930] 1958, 7, emphasis added).

4. For instance, Dirac mentions "one could carry out the energy measurement ... after such an energy measurement, it would not be possible to bring about any *interference effects* ... So as long as the photon is partly in one beam and partly in the other, *interference* can occur when the two beams are superposed, but this possibility disappears when the photon is forced entirely into one of the beams by an observation" (ibid., 8-9, emphasis added). He goes on to say "all particles can be made to exhibit interference effects and all wave motion has its energy in the form of quanta" (ibid., 10).

evident in the first statement where it are the probable states of the photon that are "interfering" and it is the outcome of this interaction that results in a specific outcome. Therefore, while attempting to show that the quantum phenomenon is unlike the classical interpretation, Dirac seems to have gradually moved from one notion to another where "interference" stands for interaction of two entities (in this context, photons). This usage of interference can be seen in some parts of Dirac's discussion.[5] This usage of "interference" as interaction comes from the classical interpretation of the phenomenon where the effect is an outcome of two waves interacting with one another.

Thus, Dirac's discussion contains two different notions of "interference" — one as a phenomenon that has specific effects (like fringe effects) and the other also indicating an interaction of two entities. However, the trouble of not separating these two creates confusion when two photons interference experiments were carried out half a century later.

6.4 Modern Interference Experiments

Post 1960s, novel kinds of interference experiments were carried out. In order to understand their novelty, some technical details about these modern experiments needs to be presented. The "conventional" interference experiments were usually conducted using thermal sources and since this kind of sources consist of numerous elementary radiators, emitting light's amplitude and phase fluctuates. Given this, light beams from two independent sources would not form the fringe pattern, which is the key aspect of interference phenomenon (Paul 1986, 211). The conventional experiments overcame this limitation by creating two beams from the same source.[6] The well known instance of this technique is the double-slit experiment (henceforth DS), where an opaque sheet with two slits or pinholes is used to create two components from a single optical source. In this experiment, the electromagnetic field at a particular point on the screen is the superposition of two fields originating from the slits. According to classical electromagnetism, the positive frequency component of the constituent electric fields can be generally expressed as —

$$E^{(+)}(r,t) = Ee^{i(kr-\omega t-\phi)} \tag{6.1}$$

5. For instance, while discussing the basic phenomenon at MZ, Dirac says "suppose we have a beam of light which is passed through some kind of interferometer, so that it gets split up into two components and the two components are subsequently made to *interfere*" (Dirac [1930] 1958, 7).

6. The emphasis on using a common source for generating the two waves in interference experiments can also be seen in Young's discussion. Kipnis, while discussing Young's theory of interference, discusses Young's knowledge of coherence and shows how Young provides different "conditions of coherence", among which one is the condition of common origin (Kipnis 1991, 125). Apart from this, the other conditions are two beams having the same frequency, having almost the same direction, presence of path difference and the light source being of a particular size. All these conditions are essentially related to the way interference was experimentally demonstrated during that time.

where E is the amplitude, (r, t) indicates a specific point in space and time, k is the wave vector, ω is the frequency and ϕ is the phase of the field (Paul 1986, 210). With this, the intensity of the superposed electric field at (r, t) on the screen can be represented in the following manner —

$$I(r,t) = {E_I}^2 + {E_{II}}^2 + 2E_I E_{II} \cos[(k_{II} - k_I)r - (\omega_{II} - \omega_I)t - (\phi_{II} - \phi_I)] \tag{6.2}$$

The equation (6.2) captures the intensity variation on the screen. When two fields of the same frequency is used, its clear from the above equation that the interference pattern varies solely on the phase difference (ibid., 210). However, the observation of fringe pattern is true only for a short duration when the phase difference remains constant. Over a period of time, the ensemble average of the intensity does not show any variation since phases keep fluctuating. In these situations, interference effect can still be identified through *intensity correlation function.* This function is the average value of the product of intensity of the superposed field at two different places (r_1, t_1) and (r_2, t_2) on the screen (ibid., 211) —

$$G^{(1)}(r_1 t_1; r_2 t_2) = \langle I(r_1, t_1) I(r_2, t_2) \rangle \tag{6.3}$$

Here, (r_1, t_1) and (r_2, t_2) are different points on the screen where the interference fringes are formed. As Paul emphasises, the averaging involved in (6.3) is the mean calculated for an ensemble (ibid., 211).[7] If the intensity of the field is considered as the square of the electric field amplitude, (6.3) would become

$$G^{(1)}(r_1 t_1; r_2 t_2) = (I_I + I_{II})^2 + 2I_I I_{II} \cos[(k_{II} - k_I)(r_2 - r_1)] \tag{6.4}$$

As (6.4) indicates, for the superposed field, $G^{(1)}(r_1 t_1; r_2 t_2)$ varies for different values of r_2 and r_1 when $t_1 = t_2$. When r_2 and r_1 mark two maximum fringe positions, then $G^{(1)}$ will be maximum. If either of these two points is situated at minimum intensity fringe position, then $G^{(1)}$ will be minimum. Therefore, when $(r_2 - r_1)$ varies in terms of fringe spacing, the correlation function for the superposed fields for specific values of r_1 and r_2 captures the intensity variation on the screen. Because of this direct correspondence, the correlation of intensities can be considered as "a reflection of the random motion of the interference pattern, and it offers an alternative procedure for establishing the existence of interference effects" (Mandel 1983, 931).

It is not that the above interpretation of interference through correlation was obvious throughout the classical times. As Bromberg points, amidst the confusion of multiple definitions of coherence, it was only in 1950s that Emil Wolf generalised the works of other

7. Paul refers the above correlation function as $G^{(2)}$ and not as $G^{(1)}$. As I will show subsequently, the equation $G^{(2)}$ would be different. Given the context of discussion, Paul should have represented the correlation function as $G^{(1)}$.

opticians and provided a definition of coherence based on correlations of field values at two distinct points (Bromberg 2016, 247). Post 1960s, the major revolutions that happened in the field of optics invariably changed the structure of interference experiments. One of the important changes here pertained to the use of laser as the source in optical experiments. Given that this novel kind of light was highly coherent, unlike the conventional experiments where a single source was used, it was possible to conduct interference experiments with two beams coming from different sources. The photodetectors also became more sensitive to pick minor and rapid fluctuations and this enabled to observe optical observations that were not possible before (Paul 1986, 209). Apart from these changes, the discovery of new kind of light brought new set of theoretical challenges. For instance, it was not clear during the initial period whether the concept of coherence and Gaussian statistics that are applicable to regular light also applies to laser (Bromberg 2016, 246-47). Another open question pertained to the HBT (Hanbury Brown and Twiss) effect which was observed using regular light, few years before these developments. There were mixed opinions about whether laser light will also show this effect (ibid., 246). The central difficulty in answering these questions had to do with the fact that there was period of uncertainty regarding the right theoretical approach to understand laser. According to Bromberg, there were three "separated traditions" during this time — classical electromagnetism, QED and quantum statistics (ibid., 244). Among them, eventually, QED established its merit over classical and semi-classical electromagnetic theories.

The availability of laser and other technical advancements made possible to conduct novel experiments that were not possible before. For instance, it was possible to experimentally explore whether two independent laser beams demonstrate interference phenomenon. Paul (1986) reviews few of the earliest "novel" interference experiments of this type. One of the first experiment mixed two optical masers in a photomultiplier and the presence of beat notes indicated the interference of these beams (Javan et al. 1962). In contrast to this indirect demonstration of interference, experiments were soon conducted where fringe pattern was successfully observed, even though intermittently, by making independent laser beams superpose within an image tube (Magyar and Mandel 1963). As was the case during the first decade of twentieth century, where experiments were conducted to investigate the possibility of interference fringes with attenuated light, similar questions were asked in the context of lasers too: "would it be possible to make two laser beams interfere even when they had been drastically attenuated"? (Paul 1986, 216). As Paul mentions, there were serious experimental difficulties to overcome for carrying out these scenarios. For instance, observation of a fringe pattern requires the presence of minimum number of photons. However, when working with attenuated laser, the exposure time would become more than the coherence time of the beams and this would result in not observing any fringe pattern. By using ingenious techniques, few experiments successfully produced this kind of phenomenon (ibid., 218-19). For instance, Pfleegor and Mandel (1967) showed the occurrence of interference of beams through anti-correlation measurement. In this experiment, attenuated laser beams from two sources were made to superpose and fall on a stack of thin glass plates (Pfleegor and Mandel 1967, 1084). The plates were arranged such that the pattern of constructive and destructive fringes form on alternate plates. Given that the light falling on all the even numbered plates go to one phototube and the light from the odd numbered plates were received in another phototube, the anticorrelation among the phototubes suggested the "presence of interference fringes". As the number of photons involved in the interference phenomenon were about 10 in every trial, a statistical approach was adopted to identify the occurrence of interference. This novel way of recognising interference phenomenon is due to the QED interpretation of radiation and a brief introduction to the redefinition of correlation function in quantum optics is necessary to understand the interference claim.

The QED interpretation of optics was initiated by Glauber. It is indeed surprising that only in 1960s that quantum mechanical interpretation of optics was carried out. As Glauber mentions "the quantum theory ... has had only a fraction of the influence upon optics that optics has historically had upon quantum theory" (Glauber 1963a, 2529). According to Glauber, this revision was delayed by the fact that classical optics was still available to give good predictions and measurements for most of the optical phenomena (ibid., 2529). QED brought various changes in the interpretation of light.[8] Here too, radiation field is understood to be composed of negative and positive frequency components

$$E(rt) = E^{(+)}(rt) + E^{(-)}(rt) \tag{6.5}$$

However, in (6.5), E, $E^{(+)}$ and $E^{(-)}$ are quantum operators. The positive and negative frequency components are photon emission and annihilation operators (ibid., 2530). This form of representation, along with the quantum mechanical machinery of operators and commutations relations, provides a different interpretation of photodetection process. For instance, Glauber mentions "although the use of the wave equation to find the field amplitude ... did not introduce any distinctions between the classical and the quantum theoretical approaches to the diffraction problem, the use of a photon counter as a detector does introduce a distinction. The photon counter is an intrinsically quantum mechanical instrument" ([1965] 2007, 26). When a detector gives a positive response, it is responding to the complex field $|\psi^{+}|^2$ and not to the intensity of the scalar field (ψ^2) (Glauber 1963a, 2531). More importantly, a positive response of the detector should be considered as part of the ensemble measurement since the theory only gives the "probability per unit time that a photon be absorbed by an ideal detector at a point r at time t" (Glauber [1965] 2007, 2531). The consideration that photodetector receives photons is quintessentially a non-classical interpretation. Because, according to the classical theory, if there are two photodetectors

8. Some of the changes were highlighted in section 5.1.

separated by a short distance, the radiation in the form of waves would trigger both of them. This kind of physical interpretation would invariably predict a paradoxical situation where each of the detectors would record one quantum of energy when the field has only one quantum of energy. Thus, only by subscribing to "corpuscular aspect of light", it is possible to claim that only one of the two detectors would be triggered and the principle of energy conservation is still respected (Paul 1986, 224). This non-classical measurement, where a photodetector situated at specific place would receive photon and not others, laid the foundation for observing delayed coincidences (Glauber 1963a, 2531).

In this new theoretical framework, Glauber generalised the notion of correlation factor. As mentioned, $G^{(1)}$ is the first-order correlation factor that measures the correlation between photodetections at two different places. In quantum mechanics, it is expressed as

$$G^{(1)}(r_1t_1; r_2t_2) = tr\{\rho E^{(-)}(r_1, t_1)E^{(+)}(r_2, t_2)\} \tag{6.6}$$

where, ρ is the density operator and tr stands for the trace of the matrices involved (ibid., 2532). A second-order correlation factor can be defined as follows —

$$G^{(2)}(r_1t_1, r_2t_2; r_3t_3, r_4t_4) = tr\{\rho E^{(-)}(r_1, t_1)E^{(-)}(r_2, t_2)E^{(+)}(r_3, t_3)E^{(+)}(r_4, t_4)\} \tag{6.7}$$

Given that correlation functions are indicators of fields' coherence, to express higher degree of coherence, higher order of correlation factors are required. As Glauber (1963a, 2534) discusses, several developments motivated this exploration. Experiments like the one conducted by Hanbury Brown and Twiss (1956) had given rise to new correlation techniques and it was necessary to understand them. Generalising $G^{(1)}$ to n-order correlation functions provides clearer interpretation of these experiments. The other important factor was with regard to the discovery of laser. Interpreting laser merely as a coherent light overlooked the "greater regularities" this kind of radiation exhibited (Glauber 1963a, 2534). To arrive at the n-order correlation function, Glauber first provides the "normalised" first-order correlation function (ibid., 2534), which is

$$g^{(1)}(r_1t_1; r_2t_2) = \frac{G^{(1)}(r_1t_1, r_2t_2)}{\{G^{(1)}(r_1t_1, r_1t_1)G^{(1)}(r_2t_2, r_2t_2)\}^{1/2}} \tag{6.8}$$

With this, he shows that the normalised n-order correlation factor — $g^{(n)}$ — can be factorised into n first-order correlation functions. This indicates that field's degree of coherence can be experimentally interpreted through the coincidences among the detections of n individual counters: a field having n-order coherence should not show statistical correlation (i.e., coincidence among photon detection) when subjected to n photodetectors (ibid., 2535).

The above reinterpretation of coherence and correlation functions brought a change in the way interference experiments were conducted. Under quantum optics, these experiment between two different sources were understood in the following generalised manner. The independent sources are considered to be constituted by specific number of atoms — M and N. The atoms in each of the sources release photons — having phase ϕ and ϕ' — when they make the transition from a higher to a lower energy state. In this scenario, the photon detection probability of a detector at a specific distance from these two sources is found to vary similar to the variation of $G^{(1)}$. Since the probability for observing the "interference fringes" depends on the phase difference $\phi - \phi'$, it is important for this difference to be constant. Otherwise, the interference term averages to zero when the experiment is conducted for a duration that is higher than coherence time (Mandel 1983, 934). This understanding of the phenomenon matches with the classical theory, which also emphasises on the phase difference being constant for observing interference. However, contrary to classical understanding, QED also provides a way to observe the "interference effects" when the phase difference randomly fluctuates (ibid., 935). In order to do so, two detectors have to be used for detecting photons at two different places. This setup makes possible to observe the second-order correlation factor $G^{(2)}$. In this way of calculating the joint probability of photons detection, the interference term is independent of phase fluctuations (ibid., 936). Pfleegor and Mandel (1967) make use of this technique to infer the occurrence of interference in their experiment.

As discussed above, one way quantum optics differs from classical optics is by showing that interference effects are not limited to observation of fringe patterns. Apart from this, quantum optics differs from the classical analysis of interference with regard to the interpretation of a scenario where the sources are two single atoms. As mentioned earlier, even when laser beams in the interference experiment are attenuated, i.e. M and N are quite small, the phenomenon observed can still be explained by classical theory. However, when there is only one atom in each source, the equivalence between quantum and classical descriptions breaks (Paul 1986, 223). As Paul mentions, when the density operator for a particular quantum state has P representation, the equivalence between the quantum mechanical and the classical descriptions holds valid (ibid., 218). Coherent light exhibit this equivalence. But, when the light source is constituted of only one excited atom, classical analysis is no more valid. This is because the joint probability of two photodetectors mentioned above shows a specific non-classical behaviour. When there are only two photons emitted, quantum mechanics dictates that the joint probability of detection is zero when the detectors are separated by an odd number of half fringes (Mandel 1983, 937). Ghosh et al. (1986) demonstrate this non-classical scenario. In this experiment, two photons are generated using parametric down conversion procedure. The generated photons — the signal and idler pair — are reflected using mirrors and are made to converge towards a narrow region. The probability of detecting one of the photons using a single detector does not indicate any modulation that suggests the occurrence of interference. However, the joint probability of using two detectors shows the interference term that does not average out to zero when the integration time is more than the coherence time (Ghosh et al. 1986, 3966). Through the normalised correlation function, which represents the interference effect in these scenarios, they successfully demonstrated the non-classical effect of two photons never being detected simultaneously when the detectors are separated by an odd number of half fringes (ibid., 3967). Given that the photons involved in this experiment were generated from the same source, this experiment still does not exhibit a genuine interference of independent photons. To supplement this shortcoming, Kaltenbaek et al. (2006) conducted an experiment using photons generated from two independent sources. Since these sources are spatially separated, the coherence properties of one does not influence the other. In this scenario too, the expected anti-correlation was observed.

6.5 Controversy about Interference of Photons

The above section summarises not only the conceptual revision of interference phenomenon with the advent of quantum optics but also the technical changes interference experiments underwent post 1960. As can be seen, the technical changes happened gradually — from independent laser beams to interference between attenuated beams and with this, finally reaching a stage where interference of two photons were performed. These novel possibilities brought several changes, not all of which sit harmoniously with the history of optics. One of the controversy that arose with the novel demonstrations pertains to Dirac's interpretation of interference. As discussed in section 6.3, Dirac's stance about interference of photons was captured in his dictum (henceforth D)

D_1 Each photon interferes only with itself

D_2 Interference between two different photons never occurs.

Until the availability of laser light, most of the interference experiments were conducted with beams emerging from a single source. This kind of experiments was understood, starting from Dirac, as exhibiting interference phenomenon that involves single-photons in quantum superposition states. It is in the context of these experiments that D was formulated. However, as I will discuss now, in the context of quantum optics and novel experiments, especially involving two independent sources, D_2 gets contested. This is because most of the experiments discussed in section 6.4 seems to be demonstrating exactly what Dirac is denying to be possible. Given this obvious conflict, several physicists, right from late 1950s to present, have responded to this situation. However, as I will show, there is no clear consensus among physicists about this: some consider that Dirac has been proven wrong; and others argue that these modern experiments further substantiate Dirac's dictum. In this section, I will summarise some of these responses. In order to do so, I will be referring to only those papers that explicitly state their opinion about Dirac's dictum. This is crucial to note because not all papers post 1960 regarding interference phenomenon involving two independent sources directly respond to Dirac's statements.

Even before the invention of laser, advent of quantum optics and the first wave of novel experiments, the confusion regarding Dirac's D_2 arose in the context of the experiments carried out by Hanbury Brown and Twiss in 1956-57. When they claimed that their experimental setup is capable of showing correlation between two photons, this inference was considered to be invalid since it violates D_2. Bromberg, while discussing this episode, cites Hanbury Brown mentioning how the HBT claim was considered by certain physicists as "patently absurd" as this claim contradicted D_2 (Bromberg 2010, 8). Hanbury Brown himself has not been consistent about the relation between HBT claim and D_2. In the 1957 paper, Hanbury Brown and Twiss clarified that their claim should not be mistaken as a claim against D_2. And, in his 1991 memoir, Hanbury Brown mentions that Dirac's statements are not applicable to the HBT experiment (ibid., 9).

Moving on, one of the earliest comments regarding the bearing of the novel interference experiments on Dirac's statements, as Bromberg notes, was from Mandel (1964). Mandel, in the introduction of his paper, distinguishes between conventional interference experiments and modern ones based on the criterion that the conventional experiments are the ones that are carried out with coherent beams. In these scenarios, classical and quantum mechanical interpretations of interference tally with one another and the interference effects can be explained through second-order field intensity. In contrast to this, the modern interference experiments are capable of showing interference effects even when beams are incoherent since quantum optics analyses interference differently (ibid., A10). This distinction is important for contextualising Mandel's comment about Dirac's statements. According to Mandel, Dirac's statements were articulated with respect to conventional experiments. Given this, it needs to be evaluated whether D is applicable in the modern context of quantum optics. Mandel is of the opinion that D_2 is not invalidated by the new findings. Like in the conventional experiments, where interference effects arises due to quantum superposition of single photons, even in the modern situation, "each photon is to be considered as being partly in both beams, and 'interferes only with itself'" (ibid., A15). Subsequently, Mandel along with Pfleegor performed the experiment discussed in section 6.4 to investigate whether interference phenomenon involving two independent laser beams "disprove" Dirac's statement (1967). In the conclusion section of their paper, they argue that the fringes are due to the superposed states associated with the detection process of each photon. With this, they conclude that Dirac's statements "appears to be as appropriate in the context of this experiment as under the more usual conditions of interferometry" (ibid., 1088).

Glauber, in his 1965 article, also comments on the confusion regarding the validity of D_2 in modern interference experiments. For him, these experiments raise "serious dilemma" only when "too great a generality to Dirac's statement" is attached (Glauber [1965] 2007, 66). Glauber mentions that Dirac's statements were meant only for experiments, like e.g. Young's DS, where single photons are involved in bringing about the interference effects. In this kind of experiments, the interference effect results from quantum superposition between alternative "evolutionary paths or histories" — the two possible spatiotemporal trajectories that single photons can take (Glauber [1965] 2007, 58). In this context, Dirac's statements are apt descriptions. However, Glauber continues, just because Dirac's articulation holds true in this scenario, it should not be considered as a "general doctrine" that is valid for all types of interference experiments (ibid., 59). To substantiate this point, Glauber considers HBT experiment which involves two photons to bring about the correlation among the detectors. In this experiment, the "alternative 'histories'" pertain not to single photon but two photons that are involved: since there are two photons and two detectors, there are two possible ways the system can evolve (ibid., 66-67). Glauber thus points that interference phenomenon in experiments like HBT is similar to experiments like DS — fringes appear in both these experiments due to quantum superposition of the possible paths. "Serious dilemma" appears only when these phenomena are articulated in terms of photons. Therefore, Glauber's response about the applicability of D_2 to the novel scenarios in quantum optics is the following: since Dirac's statements (in terms of photons) are context specific articulation of a general quantum phenomenon, considering these statements as literally true for all cases leads to interpretative problems.

Opposed to the above interpretations, Paul, in his 1986 paper about the review of interference phenomenon involving two independent sources, states at the very beginning itself that even though D_1 captures correctly the nature of the phenomenon in all the conventional interference experiments, D_2 "cannot be upheld as a general rule forbidding independent photons, i.e., photons being emitted by independent sources, to interfere" (Paul 1986, 209). For Paul, modern experiments are categorically different from conventional ones. Prior the invention of laser, since interference experiments were conducted with thermal sources, interference was brought about by using beams that are taken from a primary beam. Thus, all these conventional experiments are essentially demonstrating single photon interfering with itself. Since Dirac formulated D in this experimental context, Paul comments that "Dirac is right with respect to the experimental techniques that were at the optical researcher's disposal at that time" (ibid., 209). In contrast to these experiments, since the modern ones use beams from independent sources, "the second part of Dirac's famous statement . . . has actually been disproved" (ibid., 210). It is evident from Paul interpretation that Dirac turned out to be wrong about D_2 merely due to the technical possibilities of his time.

Among the recent works, Louradour et al. (1993) demonstrated the observation of interference fringes when two independent laser beams were superposed for a short duration. Unlike Magyar and Mandel (1963), where the fringes were unstable, the interference effect here could be observed directly by eyes. With this, they claim that D_2 is wrong. As a response to this conclusion, Wallace (1994) mentions that the inference drawn from Louradour et al. (1993) are wrong in their claim about Dirac's statement. Here, Wallace points that two photons in the concerned context are indistinguishable and it is not sensible to think a photon belongs to either of the beams since it belongs to the combined field. However, drawing from Paul (1986) and Louradour et al. (1993), Hentschel (2018) concludes that D_2 has been successfully invalidated.

Above, I have tried to summarise various opinions about how Dirac's statements fare in the context of novel experiments. Physicists post 1960s commenting on the validity of D, which was stated in late 1920s, highlight the influence of Dirac's authority in the field. Given that the modern interference experiments were completely different from the earlier ones, it was imminent to know whether these experiments challenge the earlier presumptions about interference. However, as the above discussion highlights, there is no consensus among physicists about the validity of D. Consider, for instance, the views of Glauber and Mandel. For both of them, D is quintessentially about the fundamental phenomenon that gives rise to fringe pattern: quantum superposition of states. In spite of this similarity, they disagree on their view about the validity of D. Mandel considers D is valid since interference in modern experiments still pertains to superposition of single photons. In contrast to that, Glauber suggests D is contextual and what the modern experiments emphasise is that interference arises when there are alternative histories for the whole system's evolution. Thus, even though both of them agree about the centrality of quantum superposition, each of them interpret superposition at different levels: photon level or system level.[9] Because of this distinction, Mandel and Glauber differ in their views about D_2. Paul, on the other hand, interprets D as a statement about the possible scenarios in which interference phenomenon can or cannot be observed. According to him, Dirac is suggesting that interference fringes can never be observed when there are two different, independent photons. Since the modern experiments demonstrate exactly that, he thinks D_2 has to be redacted. The varied opinions about D mentioned above highlight the controversy about interference. However, as already mentioned, in spite of multiple physicists raising differing views, the disagreement between them has not been sufficiently recognised.

6.6 Sources of Controversy

Regarding the controversy discussed above, it is important to understand the nature of the disagreement that prevails among the physicists. To begin with, the conflict between D and modern experimental observations that is found in the above views is not theoretical in nature. That is, the controversy is not about a gap in QED or the inability to theoretically

9. Interestingly, few decades later, Mandel (1983) explains the superposition at the system level.

account for a specific experimental observation. Therefore, even though two physicists might agree on QED and also about what exactly is happening in a specific experiment, they might have different definitions of "interference". This non-consensus is not something new. As I already showed in section 6.2, even in the nineteenth century, there was no clarity about how to use "superposition", "interaction" and "interference". Thus, *interference* was a fragile concept in the classical times and continues to be one even at present. If this was recognised early enough, some caution could have been exercised while using these terms and more importantly, this could have highlighted the need of conceptual exercise in clarifying these terms. Given that the situation has been overlooked, in this section, I will attempt to identify the central reasons for the controversy. I will argue that this controversy arises due to different conceptions about what constitutes as "interference" of photons. Understood in this way, the situation is nothing but a symptom of a much larger disagreement about interpretation of concepts like "interference", "different photons", "interaction", etc. In the below subsections, I will identify two specific causes of the controversy: misinterpretation of D and holding onto different definitions of "interference".

Misinterpretation of D

One of the central contentious aspects in the controversy is the interpretation of Dirac's statements. As shown in section 6.3, Dirac suggests that quantum mechanics and particle interpretation of light demand a reinterpretation of interference phenomenon. He shows that an ensemble of photons, where each of them are in superposed state, give rise to fringes. He also argues why interference should not be thought as an outcome of two photons, one in each arm of MZ, interacting with one another. Thus, by stating his dictum D he summarises his view. This set of statements has led to the present controversy. The reason for this controversy is not only circumstantial — novel experiments and observations — but also misinterpretation of Dirac's dictum. As I will show, D has been misread in two important ways: D_1 and D_2 have been considered as independent statements, and Dirac's usage of "interference" has not been analysed properly. I will illustrate these mistakes by using the views discussed in the previous section.

To begin with, consider the interpretations of D by Mandel (1964) and by Paul (1986). Mandel considers D_1 to be a positive statement which mentions what can happen and D_2 as a negative dictum. The problem arises with the interpretation of the second dictum here. Even though, D_2 is a negative statement, it needs to be clarified what exactly Dirac is denying here. Given the context of the discussion, the statements of Dirac can be paraphrased in the following way for better clarity:

${D_1}'$ *Interference* is a phenomenon which involves single photons
${D_2}'$ Interference is not due to two photons *interacting* with each other

As these rephrased statements clearly indicates, what Dirac denies is that interference should not be thought of as two photons physically interacting. When D_2 is compared with $D_2{}'$, it can be seen that Dirac uses "interference" to mean interaction in D_2. Other physicists, by granting the status of "law of interference" to Dirac's statements, have interpreted them without realising that Dirac uses "interference" in two different ways: as a phenomenon and as the physical process of interaction. So, Mandel, not paying attention to Dirac's usage, misreads D_2.

On the other hand, Paul misreads D_2 in a different way. He interprets D_2 to be denying interference phenomenon of two photons and thus considers this dictum to be invalidated by modern experiments. Here, Paul performs another misinterpretation and that is of considering D_1 and D_2 as two individual statements. For Paul, these two statements are about two different scenarios. In contrast, for Dirac, both together captures the phenomenon at MZ completely. Apart from Mandel and Paul's views, it is interesting to consider Glauber's interpretation too. In comparison with the other two, Glauber gets few aspects correct about D. He correctly recognises that D is a set of statements about earlier types of experiments and should not be generalised beyond that context. But he also overlooks that Dirac is talking about interaction, and not interference, of two photons in D_2.

The above analysis shows how a controversy like this can result from misreading of D. Apart from the above mentioned ones, there might be other reasons for this controversy. For instance, Bromberg, while discussing the history of laser, suggests difference among physicists about the "faithfulness to Dirac". He asks why is that physicists from Britain and North America felt compelled to save Dirac's dictum in comparison to other physicists who considered it to be wrong. Specifically, he points "was such faithfulness to Dirac peculiar to people trained or working in Britain? Was it true for Edward Purcell or others of the group at Harvard? Was it true here in Berlin, where a group . . . was very much involved in the problem of interference between photons from independent sources?" (Bromberg 2016, 10-11). This social and political viewpoint seems to be re-affirmed by the analysis carried out in the present section. Both Mandel and Glauber, who are American physicists, attempt to provide a positive interpretation of Dirac's statement. And contrast to them, Paul does not hesitate to conclude that Dirac has erred.

Different Criteria of Interference

After highlighting how the various interpretations of interference primarily differ from one another in their reading of D, in this sub-section I want to point another important difference about them. As I will show, these views hold onto different criteria for defining interference. The category of "interference experiments" is a big family of different kinds of experiments. This category includes varied kinds of experimental setups and observations, like for instance, DS and MZ. The summary of experiments discussed in section 6.4

mention experiments where interference is observed differently compared to the classical experiments. Given these variations, what are the criteria that qualify a phenomenon as "interference"? These criteria can be of varied kind: they may pertain to specific observations (like appearance of fringe pattern or detection correlation statistics), or validation of certain theoretical aspects (like a particular value for the correlation function) or can also be about the basic feature of the setup (like the need of two things for interacting). As I will show, there seems to be no consensus about this: for some, a phenomenon is "interference" if fringe pattern is observed; and for others, interference necessarily requires the presence of two sources. The lack of this consensus has not been recognised by the physics community. As a result of this, not only physicists are disagreeing with one another, but this has resulted in blunders where different definitions of "interference" are used within the same paper. For instance, consider Pfleegor and Mandel (1967) experiment. As the title of their paper — "Interference of Independent Photon Beams" — indicates, the experiment seems to be about the interference of independent photon beams. However, the authors conclude by theoretical situating the interference phenomenon at the level of single photons' superposition. Given this, there are two different usages of "interference" in the paper — one describing the setup of the experiment and the other explaining the phenomenon behind fringe formation. Thus, if the authors are demonstrating interference of single photons, then their title is not appropriate. Instances like this highlight the need of unpacking the criteria held by the various views and examine how they deviate from one another.

The interpretation of interference as basically a phenomenon due to quantum superposition of photons can be traced back to Dirac. As I showed in section 6.3, Dirac argues that interference is in fact an ensemble outcome of individual entities, each of which are in superposed state. This interpretation of interference is evident in the beginning of Dirac's book, where the phenomena of polarisation and interference of light are considered as case studies to illustrate the use of the new "law" — the superposition principle. For Dirac, therefore, interference of photons is nothing but "another example of superposition" ([1930] 1958, 7). If Dirac's position has to be generalised, then any phenomenon in which fringe pattern is observed due to the superposition of entities qualifies as interference. In this interpretation, it is clear when "interference" happens and when it is not. For instance, when the photons are not in superposed state — either due to their distinguishability or explicit act of probing — then fringe pattern is not observed. This criterion to invoke interference has been used by the subsequent physicists. For instance, Glauber ([1965] 2007), Pfleegor and Mandel (1967), Mandel (1983) highlight that the interference observations are due to photons being in superposed states.

Compared to the above criterion, Paul (1986) seems to be emphasising on a different criterion for interference. Given that Paul's main interest is interference experiments that involve two independent sources, he suggests that interference should be understood in a "much more general sense than in classical optics" and defines it as "superposition of electromagnetic fields, and any experiment indicating that such a superposition has taken place deserves to be called an interference experiment" (Paul 1986, 210). This definition by Paul is a completely different interpretation compared to the previous criterion. For Dirac, Glauber and others, interference phenomenon is photons being in quantum superposition states. If it is possible, even in principle, to distinguish which source the detected photon belongs to, then no fringes are observed. Opposed to this, Paul seems to be emphasising on the setup of the experiment where the fields from each source physical superpose and it is this physically superposed field which produces interference observation. Paul emphasises this at various places in the paper. For instance, while discussing experiments like that of Pfleegor and Mandel (1967) where there is only one photon at a given time, Paul mentions how naive interpretation results in paradoxical interpretation that interference is between "one photon and nothing" (Paul 1986, 221). In this kind of scenario, according to Paul, "what actually happens in that detection process is that an energy packet $h\nu$ is taken from the superposition field to which both lasers contribute equally and hence it is only natural that this photon bears information on both laser fields that becomes manifest in the ultimate interference pattern" (ibid., 221). In the conclusion of his paper, Paul reiterates that the term interference "is more generally used to denote any effect indicative of the superposition of optical fields ... Dirac's famous statement ... proves to be false" (ibid., 230). This interpretation of interference is also reflected in the way Paul interprets D.

6.7 Conclusion

As the above two sections indicate, the confusion about the relation between D and modern novel experiments arose due to the ambiguous use of certain concepts. One of the main reason that motivates this kind of misreadings is framing of D as some kind of law or principle. The usual qualification of D as "Dirac's dictum" indicates the authority of that statements. So, this formulation of D promotes non-contextual, literal interpretations of the constitutive statements such that, similar to other laws and principles, these can be applied as is in various scenarios. Apart from these reasons for the controversy, the central point of contention is also difficult to address. This is because, it is not clear what all should be considered while coming up with a definition for "interference". In any case, what this controversy clearly indicates is that we have reached a stage, which is much similar to that of Young's, where there is no clarity about how to use "interference". In this dire situation, the reasons for this controversy, which have been identified above, provide the way forward.

Chapter 7

Conclusion

The two main aims of this thesis were to analyse a specific physical entity and to show, much against the dominant perception, that philosophical analysis can meaningfully contribute to scientific practice. For this, the study of wave-particle duality of photons was found to be a suitable topic. In this concluding part of the thesis, I want to first assess the several conclusions arrived at in the previous chapters and discuss how they respond to the main aims of thesis. Later, I want to place these claims in a larger context and identify some questions that need to be further explored.

Chapter 1 of the thesis introduced various aspects of scientific entities and through this discussion, provided the motivations and scope of the study. Chapter 2 briefly highlighted the historical views and current theories about light. Given the context of the thesis topic, understanding the general notion of duality and specifically wave-particle duality were quintessential and these analyses were carried out in chapters 3 and 4. Regarding the general notion of duality, I brought forward important characteristics about it and showed how this concept can be understood better by comparing it with the concept of symmetry (Bhatta and Sarukkai 2020). Regarding wave-particle duality of radiation, I established the plurality of the concept and also argued why science should take this plurality seriously (Bhatta 2020). These novel insights and clarifications are important pre-conditions for the central enquiry I intend to analyse.

In the next two chapters, I provided important clarifications with regard to photons. In chapter 5, I argued that the duality claim about single-photons formulated using analogies cannot be substantiated. The arguments provided in this chapter positively contributes to the current scientific literature about the claim as these arguments show how to analogically interpret the concerned experimental observations. In chapter 6, I showed that interference phenomenon exhibited by photons, which is an integral part in the duality claim, is not well understood and identified few causes for this confusion. Since interference is one of the central concepts in modern theories, identifying the sources of the controversy pertaining to interference paves way for formulating coherent theory about photons.

Therefore, in order to demonstrate how philosophy can actively contribute to science, I have focused on specific phenomena — duality claim and interference — about photons and have shown philosophical analysis can provide important clarifications about the claims involved. This role played by philosophers is similar to the crucial role played by mathematicians. History of science shows ample examples were mathematicians have contributed immensely to the development and clarification of scientific theories.[1] Parallel to that, as I have shown in this thesis, philosophers can also provide insights and clarifications about scientific claims. In this sense, by addressing some challenges about photons, I have positively critiqued the reservations of naturalistic views to demarcate science and philosophy.

After having clarified how the above claims align with the larger aims of the thesis, I want to mention some of the prospective questions which need to be examined. With regard to the duality claim about single-photons, the critique presented in chapter 5 pertain to claims that involve analogies drawn in specific experiments. Along with this well known claim, there are other scenarios in which duality of photons has been articulated.[2] Since these claims are unique, context specific analysis needs to be carried out to evaluate the validity of these claims. Apart from this set of queries, another important question to explore pertains to other kinds of duality claims about photons. For instance, as discussed in sections 2.2.2 and 4.1.3, another notion of duality associated with photons is contextualised in the debate between field versus photon interpretations of modern field theories. These claims need to be examined separately to arrive at an overall understanding about the relation between the concepts duality and photons. The present study about photons also motivates an evaluation of duality claims about electrons since these are also situated in similar kinds of experiments (see Tonomura et al. (1989)).

With regard to interference phenomenon of photons, even though the discussion in chapter 6 succeeds in substantiating the controversy surrounding interference and identifies the important sources of this confusion, there are other questions which have to be addressed. Among these, the prominent one is the need of a positive definition of "interference". Discussing the various interpretations of interference and how the contemporary physicists disagree with other another identifies only the nature of the controversy. In order to completely resolve this, it is necessary to come up with criteria to decide when a phenomenon qualifies as "interference". This analysis is beyond the scope of the thesis and remains to be looked into.

1. I thank Sundar Sarukkai for suggesting this parallel between the roles of philosophers and mathematicians in the context of science.

2. The claim found in Ghose and Home (1992) is a good example here.

Bibliography

Achinstein, Peter. 1965. "The Problem of Theoretical Terms." *American Philosophical Quarterly* 2 (3): 193–203.

———. 1991. *Particles and Waves: Historical Essays in the Philosophy of Science.* Oxford: Oxford University Press.

Arabatzis, Theodore. 2011. "On the Historicity of Scientific Objects." *Erkenntnis* 75 (3): 377–390. https://doi.org/10.1007/s10670-011-9344-5.

Aspect, Alain, and Philippe Grangier. 1987. "Wave-Particle Duality for Single Photons." *Hyperfine Interactions* 37 (1-4): 1–17.

Atiyah, Sir Michael F. 2007. "Duality in Mathematics and Physics," 5:69–91. Barcelona.

Awodey, Steve. 2010. *Category Theory.* Oxford: Oxford University Press.

Bailer-Jones, Daniela M. 2009. *Scientific Models in Philosophy of Science.* University of Pittsburgh Press.

Bain, Jonathan. 2004. "Theories of Newtonian Gravity and Empirical Indistinguishability." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 35 (3): 345–376. ISSN: 1355-2198. https://doi.org/10.1016/j.shpsb.2003.10.004.

Bala, Arun. 2006. *The Dialogue of Civilizations in the Birth of Modern Science.* New York: Palgrave Macmillan.

———. 2016. *Complementarity Beyond Physics: Niels Bohr's Parallels.* Springer. ISBN: 978-3-319-39784-9.

Ballentine, Leslie E. 1998. *Quantum Mechanics: A Modern Development.* World Scientific Publishing Company. ISBN: 978-981-310-309-2.

Banerjee, Ruma. 1997. "The Yin-Yang of Cobalamin Biochemistry." *Chemistry & Biology* 4 (3): 175–186.

Bartha, Paul. 2016. "Analogy and Analogical Reasoning." In *The Stanford Encyclopedia of Philosophy,* Winter 2016, edited by Edward N. Zalta. Accessed February 12, 2018. https://plato.stanford.edu/archives/win2016/entries/reasoning-analogy.

Beller, Mara. 1992. "The Birth of Bohr's Complementarity: The Context and the Dialogues." *Studies in History and Philosophy of Science Part A* 23 (1): 147–180. https://doi.org/10.1016/0039-3681(92)90029-6.

———. 2001. *Quantum Dialogue: The Making of a Revolution.* University of Chicago Press.

Benor, Sarah Bunin, and Roger Levy. 2006. "The Chicken or the Egg? A Probabilistic Analysis of English Binomials." *Language* 82 (2): 233–278.

Bhaduri, Sadananda. 1946. *Studies in Nyāya-Vaiśeṣika Metaphysics.* Poona: Bhandarkar Oriental Research Institute.

Bhatta, Varun. 2020. "Plurality of Wave-Particle Duality." *Current Science* 118 (9): 1365–1374. https://doi.org/10.18520/cs/v118/i9/1365-1374.

Bhatta, Varun, and Sundar Sarukkai. 2020. "Duality in Science." *Current Science* 118 (5): 705–713. https://doi.org/10.18520/cs/v118/i5/705-713.

Bigaj, Tomasz. 2018. "Are Field Quanta Real Objects? Some Remarks on the Ontology of Quantum Field Theory." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 62:145–157. https://doi.org/10.1016/j.shpsb.2017.08.001.

Bogen, James. 2017. "Theory and Observation in Science." In *The Stanford Encyclopedia of Philosophy,* Summer 2017, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/sum2017/entries/science-theory-observation/.

Bogen, James, and James Woodward. 1988. "Saving the Phenomena." *The Philosophical Review* 97 (3): 303–352.

Bohm, David. 1952. "A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I." *Physical Review* 85 (2): 166–179. https://doi.org/10.1103/PhysRev.85.166.

Bohr, Niels. 1928. "The Quantum Postulate and the Recent Development of Atomic Theory." *Nature* 121:580–590. https://doi.org/10.1038/121580a0.

Bokulich, Peter. 2017. "Complementarity, Wave-Particle Duality, and Domains of Applicability." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 59:136–142.

Born, M., W. Heisenberg, and P. Jordan. (1926) 1968. "On Quantum Mechanics II." In *Sources of Quantum Mechanics,* edited by B. L. Van Der Waerden, 321–386. New York: Dover Publications.

Born, M., and P. Jordan. (1925) 1968. "On Quantum Mechanics." In *Sources of Quantum Mechanics,* edited by B. L. Van Der Waerden, 277–306. New York: Dover Publications.

Born, Max. (1926a) 1983. "On the Quantum Mechanics of Collisons." In *Quantum Theory and Measurement,* edited by John Archibald Wheeler and Wojciech Hubert Zurek, 52–55. Princeton: Princeton University Press.

———. 1926b. "Quantenmechanik Der Stoßvorgänge." *Zeitschrift für Physik* 38 (11): 803–827. ISSN: 0044-3328. https://doi.org/10.1007/BF01397184.

———. 1926c. "Zur Quantenmechanik Der Stoßvorgänge." *Zeitschrift für Physik* 37 (12): 863–867. ISSN: 0044-3328. https://doi.org/10.1007/BF01397477.

Boyer, Carl B. 1956. *History of Analytic Geometry.* New York: Scripta Mathematica.

Brading, Katherine. 2012. "Newton's Law-Constitutive Approach to Bodies: A Response to Descartes." In *Interpreting Newton: Critical Essays,* edited by Andrew Janiak and Eric Schliesser, 13–32. Cambridge: Cambridge University Press.

Brannan, David A., Matthew F. Esplen, and Jeremy J. Gray. 2011. *Geometry.* Cambridge: Cambridge University Press.

Bricmont, Jean. 2016. *Making Sense of Quantum Mechanics.* Switzerland: Springer International Publishing.

Bromberg, Joan. 1977. "Dirac's Quantum Electrodynamics and the Wave-Particle Equivalence." In *History of Twentieth Century Physics,* edited by C Weiner, 147–157. New York: Academic Press.

———. 2010. "Modelling the Hanbury Brown-Twiss Effect: The Mid-Twentieth Century Revolution in Optics." In *Third International Conference on the History of Quantum Physics.*

———. 2016. "Explaining the Laser's Light: Classical versus Quantum Electrodynamics in the 1960s." *Archive for History of Exact Sciences* 70 (3): 243–266. https://doi.org/10.1007/s00407-015-0166-8.

Brush, Stephen G. 1995. "Scientists as Historians." *Osiris* 10 (1): 214–231. https://doi.org/10.1086/368750.

Buchwald, Jed Z., and Sungook Hoon. 2003. "Physics." In *From Natural Philosophy to the Sciences: Writing the History of Nineteenth-Century Science,* edited by David Cahan, 163–195. University of Chicago Press. ISBN: 978-0-226-08927-0.

Bunge, Mario. 1968. “Analogy in Quantum Theory: From Insight to Nonsense.” *The British Journal for the Philosophy of Science* 18 (4): 265–286.

Camilleri, Kristian. 2006. “Heisenberg and the Wave–Particle Duality.” *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 37 (2): 298–315. https://doi.org/10.1016/j.shpsb.2005.08.002.

———. 2007a. “Bohr, Heisenberg and the Divergent Views of Complementarity.” *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 38 (3): 514–528. https://doi.org/10.1016/j.shpsb.2006.10.002.

———. 2007b. “Indeterminacy and the Limits of Classical Concepts: The Transformation of Heisenberg’s Thought.” *Perspectives on Science* 15 (2): 178–201. https://doi.org/10.1162/posc.2007.15.2.178.

Carpinteri, Alberto. 2014. *Structural Mechanics Fundamentals:* Florida: CRC Press.

Casse, Rey. 2006. *Projective Geometry: An Introduction.* Oxford: Oxford University Press.

Castellani, Elena. 1998. “Introduction.” In *Interpreting Bodies: Classical and Quantum Objects in Modern Physics,* edited by Elena Castellani, 3–17. New Jersey: Princeton University Press.

———. 2017. “Duality and ‘Particle’ Democracy.” *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 59:100–108.

Chakravartty, Anjan. 2003. “The Structuralist Conception of Objects.” *Philosophy of Science* 70 (5): 867–878. ISSN: 0031-8248. https://doi.org/10.1086/377373.

Cheong, Yong Wook, and Jinwoong Song. 2014. “Different Levels of the Meaning of Wave-Particle Duality and a Suspensive Perspective on the Interpretation of Quantum Theory.” *Science & Education* 23 (5): 1011–1030. ISSN: 1573-1901. https://doi.org/10.1007/s11191-013-9633-2.

Clauser, John F. 1974. “Experimental Distinction between the Quantum and Classical Field-Theoretic Predictions for the Photoelectric Effect.” *Physical Review D* 9 (4): 853–860. https://doi.org/10.1103/PhysRevD.9.853.

Cohen, I. Bernard. 1940. “The First Explanation of Interference.” *American Journal of Physics* 8 (2): 99–106. https://doi.org/10.1119/1.1991547.

Cohen-Tannoudji, Claude. 1991. *Quantum Mechanics.* Vol. 1. Wiley. ISBN: 978-0-471-16433-3.

Colwell, Robert K., and Thiago F. Rangel. 2009. “Hutchinson’s Duality: The Once and Future Niche.” *Proceedings of the National Academy of Sciences* 106:19651–19658. https://doi.org/10.1073/pnas.0901650106.

Combourieu, Marie Christine, and Helmut Rauch. 1992. "The Wave-Particle Dualism in 1992: A Summary." *Foundations of Physics* 22 (12): 1403–1434. https://doi.org/10.1007/BF01883732.

Compton, A. H. 1929. "The Corpuscular Properties of Light." *Reviews of Modern Physics* 1 (1): 74–89. https://doi.org/10.1103/RevModPhys.1.74.

Corfield, David. 2017. "Duality as a Category-Theoretic Concept." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 59:55–61.

Coxeter, H.S.M. 1987. *Projective Geometry.* New York: Springer-Verlag.

———. 1993. *The Real Projective Plane.* New York: Springer-Verlag.

Cushing, James T. 1994. *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony.* University of Chicago Press.

———. 1996. "The Causal Quantum Theory Program." In *Bohmian Mechanics and Quantum Theory: An Appraisal,* edited by James T. Cushing, Arthur Fine, and Sheldon Goldstein, 1–20. Dordrecht: Springer Science & Business Media.

Darrigol, Olivier. 1986. "The Origin of Quantized Matter Waves." *Historical Studies in the Physical and Biological Sciences* 16 (2): 197–253. https://doi.org/10.2307/27757565.

———. 1994. "The Electron Theories of Larmor and Lorentz: A Comparative Study." *Historical Studies in the Physical and Biological Sciences* 24 (2): 265–336. https://doi.org/10.2307/27757725.

———. 2012. *A History of Optics from Greek Antiquity to the Nineteenth Century.* Oxford University Press.

Daston, Lorraine. 2000a. "Preternatural Philosophy." In *Biographies of Scientific Objects,* edited by Lorraine Daston, 15–41. University of Chicago Press.

———. 2000b. "The Coming Into Being of Scientific Objects." In *Biographies of Scientific Objects,* edited by Lorraine Daston, 1–14. University of Chicago Press.

De Broglie, Louis. (1924) 2004. *On the Theory of Quanta.* Translated by A. F. Kracklauer. Paris: Foundation Louis de Broglie.

———. (1925) 1963. *Recherches Sur La Theorie Des Quanta.* Paris: Masson.

De Regt, Henk W. 2017. *Understanding Scientific Understanding.* Oxford University Press.

Dewdney, C., G. Horton, M. M. Lam, Z. Malik, and M. Schmidt. 1992. "Wave-Particle Dualism and the Interpretation of Quantum Mechanics." *Foundations of Physics* 22 (10): 1217–1265. https://doi.org/10.1007/BF01889712.

Dewdney, Chris, and George Horton. 1996. "De Broglie, Bohm and the Boson." In *Bohmian Mechanics and Quantum Theory: An Appraisal,* edited by J. T. Cushing, Arthur Fine, and S. Goldstein, 169–190. Dordrecht: Springer Science & Business Media. ISBN: 978-94-015-8715-0.

Dieudonné, Jean. 1981. *History of Functional Analysis.* Amsterdam: North-Holland Publishing Company.

Dimitrova, T. L., and A. Weis. 2008. "The Wave-Particle Duality of Light: A Demonstration Experiment." *American Journal of Physics* 76 (2): 137–142.

Dirac, P. A. M. 1927. "The Quantum Theory of the Emission and Absorption of Radiation." *Proceedings of the Royal Society of London. Series A* 114 (767): 243–265. https://doi.org/10.1098/rspa.1927.0039.

———. (1930) 1958. *The Principle of Quantum Mechanics.* Fourth. Oxford: Clarendon Press.

Dongen, Jeroen Van. 2007. "The Interpretation of the Einstein-Rupp Experiments and Their Influence on the History of Quantum Mechanics." *Historical Studies in the Physical and Biological Sciences* 37 (suppl): 121–131.

DuMond, Jesse W. M. 1938. "The "Palace of Discovery" at the Paris Exposition of 1937." *Journal of Applied Physics* 9 (5): 289–294. https://doi.org/10.1063/1.1710418.

Duncan, Anthony, and Michel Janssen. 2008. "Pascual Jordan's Resolution of the Conundrum of the Wave-Particle Duality of Light." 39 (3): 634–666. https://doi.org/10.1016/j.shpsb.2008.04.005.

Einstein, Albert. (1905a) 1965. "Concerning an Heuristic Point of View Toward the Emission and Transformation of Light." Translated by A. B. Arons and M. B. Peppard. *American Journal of Physics* 33 (5): 367–374. https://doi.org/10.1119/1.1971542.

———. (1905b) 1989. "On the Electrodynamics of Moving Bodies." In *The Collected Papers of Albert Einstein: The Swiss Years Writings, 1900-1909,* translated by Anna Beck, 140–171. New Jersey: Princeton University Press.

———. (1909) 1989. "On the Development of Our Views Concerning the Nature and Constitution of Radiation." In *The Collected Papers of Albert Einstein: The Swiss Years Writings, 1900-1909,* translated by Anna Beck, 379–394. New Jersey: Princeton University Press.

Ekspong, Gösta. 1999. "The Dual Nature of Light, as Reflected in the Nobel Archive." *Proceedings of the American Philosophical Society* 143 (1): 42–49.

Ellett, A., H. F. Olson, and H. A. Zahl. 1929. "The Reflection of Atoms from Crystals." *Physical Review* 34 (3): 493–501. https://doi.org/10.1103/PhysRev.34.493.

Falconer, Isobel. 1987. "Corpuscles, Electrons and Cathode Rays: J.J. Thomson and the 'Discovery of the Electron'." *The British Journal for the History of Science* 20 (3): 241–276. https://doi.org/10.1017/S0007087400023955.

Falk, Raphael. 1986. "What Is a Gene?" *Studies in History and Philosophy of Science Part A* 17 (2): 133–173. ISSN: 0039-3681. https://doi.org/10.1016/0039-3681(86)90024-5.

Falkenburg, Brigette. 2007. *Particle Metaphysics: A Critical Account of Subatomic Reality.* Springer.

Fermi, Enrico. 1932. "Quantum Theory of Radiation." *Reviews of Modern Physics* 4 (1): 87–132. https://doi.org/10.1103/RevModPhys.4.87.

Feynman, R. P., R. B. Leighton, and M. Sands. 2010. *The Feynman Lectures on Physics.* Vol. III. New York: Basic Books.

Fick, Dieter, and Horst Kant. 2009. "Walther Bothe's Contributions to the Understanding of the Wave-Particle Duality of Light." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 40 (4): 395–405. https://doi.org/10.1016/j.shpsb.2009.08.005.

Fine, Arthur. 1996. "On the Interpretation of Bohmian Mechanics." In *Bohmian Mechanics and Quantum Theory: An Appraisal,* edited by J. T. Cushing, Arthur Fine, and S. Goldstein, 231–250. Dordrecht: Springer Science & Business Media. ISBN: 978-94-015-8715-0.

Fox, Mark. 2006. *Quantum Optics: An Introduction.* Oxford: Oxford University Press.

Fraser, Doreen. 2008. "The Fate of 'Particles' in Quantum Field Theories with Interactions." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 39 (4): 841–859. https://doi.org/10.1016/j.shpsb.2008.05.003.

French, Steven. 1998. "On the Withering Away of Physical Objects." In *Interpreting Bodies: Classical and Quantum Objects in Modern Physics,* edited by Elena Castellani, 93–113. New Jersey: Princeton University Press.

———. 2010. "The Interdependence of Structure, Objects and Dependence." *Synthese* 175 (1): 89–109. https://doi.org/10.1007/s11229-010-9734-2.

———. 2014. *The Structure of the World: Metaphysics and Representation.* Oxford: Oxford University Press.

French, Steven, and Décio Krause. 2006. *Identity in Physics: A Historical, Philosophical, and Formal Analysis.* Oxford: Clarendon Press.

Garber, Daniel. 1992. *Descartes' Metaphysical Physics.* Chicago: The University of Chicago Press.

———. 2006. "Physics and Foundations." In *The Cambridge History of Science: Early Modern Science,* edited by Katharine Park and Lorraine Daston, 3:21–69. Cambridge: Cambridge University Press.

Ghose, Partha, and Dipankar Home. 1992. "Wave-Particle Duality of Single-Photon States." *Foundations of Physics* 22 (12): 1435–1447.

Ghosh, R., C. K. Hong, Z. Y. Ou, and L. Mandel. 1986. "Interference of Two Photons in Parametric down Conversion." *Physical Review A* 34 (5): 3962–3968. https://doi.org/10.1103/PhysRevA.34.3962.

Gingras, Yves. 2001. "What Did Mathematics Do to Physics?" *History of Science* 39 (4): 383–416. ISSN: 0073-2753. https://doi.org/10.1177/007327530103900401.

Glauber, Roy J. 1963a. "Coherent and Incoherent States of the Radiation Field." *Physical Review* 131 (6): 2766–2788. https://doi.org/10.1103/PhysRev.131.2766.

———. 1963b. "Photon Correlations." *Physical Review Letters* 10, no. 3 (February 1, 1963): 84–86. Accessed July 19, 2019. https://doi.org/10.1103/PhysRevLett.10.84. https://link.aps.org/doi/10.1103/PhysRevLett.10.84.

———. 1963c. "The Quantum Theory of Optical Coherence." *Physical Review* 130 (6): 2529–2539.

———. (1965) 2007. "Optical Coherence and Photon Statistics." In *Quantum Theory of Optical Coherence: Selected Papers and Lectures,* 23–182. Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA.

Gracia, Jorge J. E. 1988. *Individuality: An Essay on The Foundations of Metaphysics.* Albany: State University of New York Press.

Grangier, Philippe, and Izo Abram. 2003. "Single Photons on Demand." *Physics World* 16 (2): 31–35. https://doi.org/10.1088/2058-7058/16/2/35.

Grangier, Philippe, Gerard Roger, and Alain Aspect. 1986. "Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences." *Europhysics Letters* 1 (4): 173–179.

Greca, Ileana M., and Olival Freire. 2014. "Meeting the Challenge: Quantum Physics in Introductory Physics Courses." In *International Handbook of Research in History, Philosophy and Science Teaching,* edited by Michael R. Matthews, 183–209. Dordrecht: Springer Netherlands. ISBN: 978-94-007-7654-8.

Griffiths, D. J. 2005. *Introduction to Quantum Mechanics.* Second edition. New Jersey: Pearson Education International.

Hacking, Ian. 1983. *Representing and Intervening: Introductory Topics in the Philosophy of Natural Science.* Cambridge: Cambridge University Press.

Halbfass, Wilhelm. 1992. *On Being and What There Is: Classical Vaisesika and the History of Indian Ontology.* SUNY Press. ISBN: 978-0-7914-1178-0.

Halvorson, Hans, and Rob Clifton. 2002. "No Place for Particles in Relativistic Quantum Theories?" *Philosophy of Science* 69 (1): 1–28.

Hanbury Brown, R., and R. Q. Twiss. 1956. "Correlation between Photons in Two Coherent Beams of Light." *Nature* 177:27–29.

———. 1957. "Interferometry of the Intensity Fluctuations in Light. I. Basic Theory: The Correlation between Photons in Coherent Beams of Radiation." *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* 242 (1230): 300–324.

Hanson, N. R. 1963. "The Dematerialization of Matter." In *The Concept of Matter,* edited by Ernan McMullin, 549–561. Notre Dame, Indiana: University of Notre Dame Press.

Hanson, Norwood Russell. 1961. "Discovering the Positron (i)." *The British Journal for the Philosophy of Science* XII (47): 194–214. https://doi.org/10.1093/bjps/XII.47.194.

Hawley, Katherine. 2018. "Temporal Parts." In *The Stanford Encyclopedia of Philosophy,* Spring 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2018/entriesemporal-parts/.

Heisenberg, W. 1929. "Die Entwicklung Der Quantentheorie 1918–1928." *Naturwissenschaften* 17 (26): 490–496. https://doi.org/10.1007/BF01505682.

———. (1930) 1949. *The Physical Principles of the Quantum Theory.* Translated by Carl Eckart and Frank Hoyt. Toronto: Dover Publications.

Held, Carsten. 1994. "The Meaning of Complementarity." *Studies in History and Philosophy of Science Part A* 25 (6): 871–893.

Hendry, John. 1980. "The Development of Attitudes to the Wave-Particle Duality of Light and Quantum Theory, 1900–1920." *Annals of Science* 37:59–79.

Hentschel, Klaus. 2018. *Photons: The History and Mental Models of Light Quanta.* Springer International Publishing AG.

Hesse, Mary B. 1970. *Models and Analogies in Science.* Indiana: University of Notre Dame Press.

Hill, E. L. 1936. "Recent Work on the Compton Effect." *Review of Scientific Instruments* 7 (6): 225–228. https://doi.org/10.1063/1.1752132.

Hirosige, Tetu. 1976. "The Ether Problem, the Mechanistic Worldview, and the Origins of the Theory of Relativity." *Historical Studies in the Physical Sciences* 7:3–82.

Hobson, Art. 2013. "There Are No Particles, There Are Only Fields." *American Journal of Physics* 81 (3): 211–223.

Holyoak, Keith J., and Paul Thagard. 1995. *Mental Leaps: Analogy in Creative Thought.* Massachusetts: MIT Press. ISBN: 978-0-262-58144-8.

Home, Dipankar. 1997. *Conceptual Foundations of Quantum Physics: An Overview from Modern Perspectives.* New York: Springer Science & Business Media.

Hon, Giora, and Bernard Goldstein. 2008. *From Summetria to Symmetry: The Making of a Revolutionary Scientific Concept.* Springer.

Horn, Laurence R., and Heinrich Wansing. 2017. "Negation." In *The Stanford Encyclopedia of Philosophy,* Spring 2017, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2017/entries/negation/.

Hughes, R. I. G. 1989. *The Structure and Interpretation of Quantum Mechanics.* Harvard University Press. ISBN: 978-0-674-84392-9.

Hyde, Dominic, and Diana Raffman. 2018. "Sorites Paradox." In *The Stanford Encyclopedia of Philosophy,* Summer 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/sum2018/entries/sorites-paradox/.

Hylton, Peter. 2007. *Quine.* New York: Routledge.

Jammer, Max. 1966. *The Conceptual Development of Quantum Mechanics.* McGraw-Hill Book Company.

———. 1997. *Concepts of Mass in Classical and Modern Physics.* New York: Dover Publication.

———. 2012. *Concepts of Force: A Study in the Foundations of Dynamics.* Dover Publications. ISBN: 978-1-306-36267-2.

———. 2013. *Concepts of Space: The History of Theories of Space in Physics.* Dover Publications. ISBN: 978-1-306-39077-4.

Javan, A., E. A. Ballik, and W. L. Bond. 1962. "Frequency Characteristics of a Continuous-Wave He–Ne Optical Maser." *Journal of the Optical Society of America* 52 (1): 96–98. https://doi.org/10.1364/JOSA.52.000096.

Jeans, J. H. 1910. "On Non-Newtonian Mechanical Systems, and Planck's Theory of Radiation." *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 20 (120): 943–954.

Kaltenbaek, Rainer, Bibiane Blauensteiner, Marek Żukowski, Markus Aspelmeyer, and Anton Zeilinger. 2006. "Experimental Interference of Independent Photons." *Physical Review Letters* 96 (24): 240502. https://doi.org/10.1103/PhysRevLett.96.240502.

Kidd, Richard, James Ardini, and Anatol Anton. 1989. "Evolution of the Modern Photon." *American Journal of Physics* 57 (1): 27–35.

Kipnis, Nahum. 1991. *History of the Principle of Interference of Light.* Basel: Springer Basel AG.

Klein, Martin J. 1961. "Max Planck and the Beginnings of the Quantum Theory." *Archive for History of Exact Sciences* 1 (5): 459–479.

———. 1963. "Einstein's First Paper on Quanta." *The Natural Philosopher* 2:59–86.

———. 1964. "Einstein and the Wave-Particle Duality." *The Natural Philosopher* 3:3–49.

———. 1985. *Paul Ehrenfest: The Making of a Theoretical Physicist.* Vol. 1. Amsterdam: North-Holland Physics Publishing.

Kojevnikov, Alexei. 2002. "Einstein's Fluctuation Formula and the Wave-Particle Duality." In *Einstein Studies in Russia,* edited by Yuri Balashov and Vladimir Vizgin, 10:181–228. Einstein Studies. Boston: Birkhäuser.

Kolesov, Roman, Bernhard Grotz, Gopalakrishnan Balasubramanian, Rainer J. Stöhr, Aurélien A. L. Nicolet, Philip R. Hemmer, Fedor Jelezko, and Jörg Wrachtrup. 2009. "Wave–Particle Duality of Single Surface Plasmon Polaritons." *Nature Physics* 5 (7): 470–474. https://doi.org/10.1038/nphys1278.

Kragh, Helge. 1990. *Dirac: A Scientific Biography.* Cambridge: Cambridge University Press.

Kreyszig, Erwin. 1978. *Introductory Functional Analysis with Application.* New York: John Wiley & Sons.

Kroon, Fred, and Alberto Voltolini. 2018. "Fictional Entities." In *The Stanford Encyclopedia of Philosophy,* Winter 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2018/entries/fictional-entities/.

Kuhlmann, Meinard. 2015. "Quantum Field Theory." In *The Stanford Encyclopedia of Philosophy,* Summer 2015, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. Accessed March 30, 2018. https://plato.stanford.edu/archives/sum2015/entries/quantum-field-theory/.

Kuhn, Thomas S. 1976. "Mathematical vs. Experimental Traditions in the Development of Physical Science." *The Journal of Interdisciplinary History* 7 (1): 1–31. ISSN: 0022-1953. https://doi.org/10.2307/202372.

Ladyman, James. 2016. "Structural Realism." In *The Stanford Encyclopedia of Philosophy,* Winter 2016, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2016/entries/structural-realism/.

Ladyman, James, Don Ross, David Spurrett, and John Collier. 2007. *Every Thing Must Go: Metaphysics Naturalized.* Oxford University Press. ISBN: 978-0-19-927619-6.

Lamb, W. E. 1995. "Anti-Photon." *Applied Physics B* 60 (2-3): 77–84.

Landau, L. D., and E. M. Lifshitz. 1977. *Quantum Mechanics: Non-Relativistic Theory.* Third edition. Oxford: Pergamon Press.

Landé, Alfred. 1959. "From Dualism to Unity in Quantum Mechanics." *The British Journal for the Philosophy of Science* 10 (37): 16–24.

———. 1962. "The Case against Quantum Duality." *Philosophy of Science* 29 (1): 1–6.

———. 1971. "The Decline and Fall of Quantum Dualism." *Philosophy of Science* 38 (2): 221–223.

Latchford, Kenneth Arthur. 1975. "Thomas Young and the Evolution of the Interference Principle," University of London.

Leinster, T. 2014. *Basic Category Theory.* Cambridge: Cambridge University Press.

Lewis, Gilbert N. 1926. "The Conservation of Photons." *Nature* 118 (2981): 874. https://doi.org/10.1038/118874a0.

Loudon, Rodney. 1980. "Non-Classical Effects in the Statistical Properties of Light." *Reports on Progress in Physics* 43 (7): 913–949.

———. 2001. *The Quantum Theory of Light.* Third Edition. Oxford: Oxford University Press.

———. 2008. "What Is a Photon?" In *The Nature of Light: What Is a Photon?,* edited by Chandra Roychoudhuri, A. F. Kracklauer, and Katherine Creath, 11–21. Boca Raton: CRC Press.

Lounis, Brahim, and Michel Orrit. 2005. “Single-Photon Sources.” *Reports on Progress in Physics* 68 (5): 1129–1179.

Louradour, F., F. Reynaud, B. Colombeau, and C. Froehly. 1993. “Interference Fringes between Two Separate Lasers.” *American Journal of Physics* 61 (3): 242–245. https://doi.org/10.1119/1.17298.

Lowe, E. J. 1998. *The Possibility of Metaphysics: Substance, Identity, and Time.* Oxford University Press. ISBN: 0-19-151914-6.

———. 2003. “Individuation.” In *The Oxford Handbook of Metaphysics,* edited by Michael J. Loux and Dean W. Zimmerman, 75–95. New York: Oxford University Press.

MacLane, Saunders. 1950. “Duality for Groups.” *Bulletin of the American Mathematical Society* 56 (6): 485–516.

Magyar, G, and L. Mandel. 1963. “Interference Fringes Produced by Superposition of Two Independent Maser Light Beams.” *Nature* 198 (4877): 255–256.

Malament, David B. 1996. “In Defense of Dogma: Why There Cannot Be a Relativistic Quantum Mechanics of (Localizable) Particles.” In *Perspectives on Quantum Reality,* edited by Rob Clifton, 1–10. Springer Netherlands. https://doi.org/10.1007/978-94-015-8656-6_1.

Mandel, L. 1964. “Quantum Theory of Interference Effects Produced by Independent Light Beams.” *Physical Review* 134 (1A): A10–A15. https://doi.org/10.1103/PhysRev.134.A10.

———. 1983. “Photon Interference and Correlation Effects Produced by Independent Quantum Sources.” *Physical Review A* 28 (2): 929–943. https://doi.org/10.1103/PhysRevA.28.929.

Matilal, Bimal Krishna. 1986. *Perception: An Essay on Classical Indian Theories of Knowledge.* Oxford: Clarendon Press.

———. 1990. *Logic, Language, and Reality: Indian Philosophy and Contemporary Issues.* Delhi: Motilal Banarsidass Publishers.

Maxwell, Grover. 1962. “The Ontological Status of Theoretical Entities.” In *Scientific Explanation, Space and Time,* edited by H. Feigel and Grover Maxwell, 3–27. Minneapolis: University of Minnesota Press.

Maxwell, Nicholas. 1988. “Quantum Propensiton Theory: A Testable Resolution of the Wave/Particle Dilemma.” *The British Journal for the Philosophy of Science* 39 (1): 1–50.

McCafferty, Dewey G, Predrag Cudic, Michael K Yu, Douglas C Behenna, and Ryan Kruger. 1999. “Synergy and Duality in Peptide Antibiotic Mechanisms.” *Current Opinion in Chemical Biology* 3 (6): 672–680.

McCormmach, Russell. 1967. “J. J. Thomson and the Structure of Light.” *The British Journal for the History of Science* 3 (4): 362–387.

———. 1970a. “Einstein, Lorentz, and the Electron Theory.” *Historical Studies in the Physical Sciences* 2:41–87. https://doi.org/10.2307/27757304.

———. 1970b. “H. A. Lorentz and the Electromagnetic View of Nature.” *Isis* 61 (4): 459–497. https://doi.org/10.1086/350681.

McMullin, Ernan. 1963. “Introduction.” In *The Concept of Matter,* edited by Ernan McMullin, 1–41. Notre Dame, Indiana: University of Notre Dame Press.

———. 1978. *Newton on Matter and Activity.* Notre Dame, Indiana: University of Notre Dame Press.

———. 1985. “Galilean Idealization.” *Studies in History and Philosophy of Science Part A* 16 (3): 247–273. https://doi.org/10.1016/0039-3681(85)90003-2.

———. 2010. “From Matter to Materialism...and (Almost) Back.” In *Information and the Nature of Reality: From Physics to Metaphysics,* edited by Paul Davies and Niels Henrik Gregersen. Cambridge University Press. ISBN: 978-1-139-49096-2.

Milonni, P. W. 1984. “Wave-Particle Duality of Light: A Current Perspective.” In *The Wave-Particle Dualism,* edited by S. Diner, D. Fargue, G. Lochak, and F. Selleri, 27–68. Dordrecht: D. Reidel Publishing Company.

Mittelstaedt, Peter. 1998. “The Constitution of Objects in Kant’s Philosophy and in Modern Physics.” In *Interpreting Bodies: Classical and Quantum Objects in Modern Physics,* edited by Elena Castellani, 168–180. New Jersey: Princeton University Press.

Mollon, J. D. 2002. “The Origins of the Concept of Interference.” *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 360 (1794): 807–819. https://doi.org/10.1098/rsta.2001.0968.

Moore, A. W. 2012. *The Evolution of Modern Metaphysics: Making Sense of Things.* Cambridge University Press. ISBN: 978-0-521-61655-3.

Nagel, Ernest. 1939. “The Formation of Modern Conceptions of Formal Logic in the Development of Geometry.” *Osiris* 7:142–223.

Navarro, Luis, and Enric Pérez. 2004. “Paul Ehrenfest on the Necessity of Quanta (1911): Discontinuity, Quantization, Corpuscularity, and Adiabatic Invariance.” *Archive for History of Exact Sciences* 58 (2): 97–141.

Nelson, Michael. 2019. "Existence." In *The Stanford Encyclopedia of Philosophy,* Spring 2019, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2019/entries/existence/.

Noonan, Harold, and Ben Curtis. 2018. "Identity." In *The Stanford Encyclopedia of Philosophy,* Summer 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. Accessed December 19, 2019. https://plato.stanford.edu/archives/sum2018/entries/identity/.

Norton, John D. 2000. "How We Know about Electrons." In *After Popper, Kuhn and Feyerabend,* edited by Robert Nola and Howard Sankey, 67–97. Dordrecht: Springer Science & Business Media.

———. 2006. "Atoms, Entropy, Quanta: Einstein's Miraculous Argument of 1905." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 37 (1): 71–100.

Omnès, Roland. 1994. *The Interpretation of Quantum Mechanics.* New Jersey: Princeton University Press.

Oudshoorn, Nelly. 1990. "Endocrinologists and the Conceptualization of Sex, 1920–1940." *Journal of the History of Biology* 23 (2): 163–186.

Pais, Abraham. 1986. *Inward Bound: Of Matter and Forces in the Physical World.* Oxford University Press.

Papineau, David. 2016. "Naturalism." In *The Stanford Encyclopedia of Philosophy,* Winter 2016, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2016/entries/naturalism/.

Paul, H. 1986. "Interference between Independent Photons." *Reviews of Modern Physics* 58 (1): 209–231.

Pedoe, Dan. 1975. "Notes on the History of Geometrical Ideas II. The Principle of Duality." *Mathematics Magazine* 48 (5): 274–277.

Peelo, David F. 2014. "Appendix B: Principle of Duality." In *Current Interruption Transients Calculation,* West Sussex, 187–189. John Wiley & Sons Ltd.

Perillán, José G. 2018. "Quantum Narratives and the Power of Rhetorical Omission: An Early History of the Pilot Wave Interpretation of Quantum Theory." *Historical Studies in the Natural Sciences* 48 (1): 24–55. ISSN: 1939-1811. https://doi.org/10.1525/hsns.2018.48.1.24.

Pfleegor, R. L., and L. Mandel. 1967. "Interference of Independent Photon Beams." *Physical Review* 159 (5): 1084–1088. https://doi.org/10.1103/PhysRev.159.1084.

Phillips, A. C. 2013. *Introduction to Quantum Mechanics.* John Wiley & Sons, May 20, 2013. ISBN: 978-1-118-72325-8.

Phillips, Stephen H. 1997. *Classical Indian Metaphysics: Refutations of Realism and the Emergence of "New Logic.* Motilal Banarsidass Publ. ISBN: 978-81-208-1489-9.

Pipkin, Francis M. 1979. "Atomic Physics Tests of the Basic Concepts in Quantum Mechanics." *Advances in Atomic and Molecular Physics* 14:281–340. https://doi.org/10.1016/S0065-2199(08)60130-X.

Planck, Max. (1900a) 1967. "On an Improvement of Wien's Equation for the Spectrum." In *The Old Quantum Theory,* edited by D. Ter Harr, 79–81. Oxford: Pergamon Press.

———. (1900b) 1967. "On the Theory of the Energy Distribution Law of the Normal Spectrum." In *The Old Quantum Theory,* edited by D. Ter Harr, 82–90. Oxford: Pergamon Press.

Polchinski, Joseph. 2017. "Dualities of Fields and Strings." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 59:6–20.

Psillos, Stathis. 2005. *Scientific Realism: How Science Tracks Truth.* Routledge.

Puccetti, Roland. 1989. "Two Brains, Two Minds? Wigan's Theory of Mental Duality." *The British Journal for the Philosophy of Science* 40 (2): 137–144.

Purcell, E. M. 1956. "The Question of Correlation between Photons in Coherent Light Rays." *Nature* 178:1449–1450.

Quine, W. V. 1956. *Methods of Logic.* New York: Holt, Rinehart and Winston.

———. (1960) 2013. *Word and Object.* Massachusetts: MIT Press.

———. 1963. "On What There Is." In *From a Logical Point of View,* 1–20. New York: Harper Torchbooks.

———. 1969. "Epistemology Naturalized." In *Ontological Relativity and Other Essays,* 69–90. Columbia University Press.

———. 1976. "Whither Physical Objects?" In *Essays in the Memory of Imre Lakatos,* edited by R. S. Cohen, P. K. Feyerabend, and M. W. Wartofsky, 497–504. Dordrecht: D. Reidel Publishing Company.

———. 1978. "Facts of the Matter." *The Southwestern Journal of Philosophy* 9 (2): 155–169.

———. 1982a. "On the Individuation of Attributes." In *Theories and Things,* 100–112. Massachusetts: The Belknap Press of Harvard University Press.

Quine, W. V. 1982b. “Things and Their Place in Theories.” In *Theories and Things,* 1–23. Massachusetts: The Belknap Press of Harvard University Press.

Ramsey, William. 2019. “Eliminative Materialism.” In *The Stanford Encyclopedia of Philosophy,* Spring 2019, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2019/entries/materialism-eliminative/.

Redhead, Michael. 1987. *Incompleteness, Nonlocality, and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics.* Oxford: Clarendon Press.

Richardson, Alan. 1997. “Toward a History of Scientific Philosophy.” *Perspectives on Science* 5 (3): 418–451.

Robinson, Howard. 2018. “Substance.” In *The Stanford Encyclopedia of Philosophy,* Winter 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2018/entries/substance/.

Rosen, Gideon. 2018. “Abstract Objects.” In *The Stanford Encyclopedia of Philosophy,* Winter 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2018/entries/abstract-objects/.

Sabra, A. I. 1981. *Theories of Light: From Descartes to Newton.* Cambridge: Cambridge University Press.

Sakurai, J. J., and Jim Napolitano. 2011. *Modern Quantum Mechanics.* Second edition. San Francisco: Addison-Wesely.

Sarkar, Benoy Kumar. 1918. *Hindu Achievements in Exact Science: A Study in the History of Scientific Development.* Bombay: Longmans, Green and Co.

Sarukkai, Sundar. 2014. “Indian Experiences with Science: Considerations for History, Philosophy, and Science Education.” In *International Handbook of Research in History, Philosophy and Science Teaching,* edited by Michael R. Matthews, 1691–1719. Dordrecht: Springer Netherlands. ISBN: 978-94-007-7654-8.

Scarani, Valerio, and Antoine Suarez. 1998. “Introducing Quantum Mechanics: One-Particle Interferences.” *American Journal of Physics* 66 (8): 718–721.

Schiff, Leonard I. 1955. *Quantum Mechanics.* McGraw-Hill Book Company.

Schweber, Silvan S. 1994. *QED and the Men Who Made It: Dyson, Feynman, Schwinger and Tomonaga.* Princeton: Princeton University Press.

Scully, Marlan O., and Murray Sargent. 1972. “The Concept of the Photon.” *Physics Today* 25 (3): 38–47.

Scully, Marlan O., and Herbert Walther. 1989. "Quantum Optical Test of Observation and Complementarity in Quantum Mechanics." *Physical Review A* 39 (10): 5229–5236. https://doi.org/10.1103/PhysRevA.39.5229.

Scully, Marlan O., and M. Suhail Zubairy. 2001. *Quantum Optics.* Cambridge: Cambridge University Press.

Seal, Brajendranath. 1915. *The Positive Sciences of the Ancient Hindus.* Bombay: Longmans, Green and Co.

Seth, Suman. 2004. "Quantum Theory and the Electromagnetic World-View." *Historical Studies in the Physical and Biological Sciences* 35 (1): 67–93. https://doi.org/10.1525/hsps.2004.35.1.67.

Shankar, R. 1994. *Principles of Quantum Mechanics.* Plenum Press.

Shapere, Dudley. 1982. "The Concept of Observation in Science and Philosophy." *Philosophy of Science* 49 (4): 485–525. https://doi.org/10.1086/289075.

Shapiro, Alan E. 2004. "Newton's Optics and Atomism." In *The Cambridge Companion to Newton,* edited by Bernard I Cohen and George E Smith, 227–255. Cambridge: Cambridge University Press.

Shelley, Cameron. 2002. "Analogy Counterarguments and the Acceptability of Analogical Hypotheses." *The British Journal for the Philosophy of Science* 53 (4): 477–496. https://doi.org/10.1093/bjps/53.4.477.

Shields, Christopher John. 2013. *Aristotle.* Routledge. ISBN: 978-0-415-62249-3.

Shrader-Frechette, K. S. 1980. "Recent Changes in the Concept of Matter: How Does 'Elementary Particle' Mean?" *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 1980 (1): 302–316. ISSN: 0270-8647. https://doi.org/10.1086/psaprocbienmeetp.1980.1.192574.

Silva, Indianara, and Olival Freire. 2013. "The Concept of the Photon in Question: The Controversy Surrounding the HBT Effect circa 1956–1958." *Historical Studies in the Natural Sciences* 43 (4): 453–491.

Stoljar, Daniel. 2010. *Physicalism.* Oxford: Routledge.

———. 2017. "Physicalism." In *The Stanford Encyclopedia of Philosophy,* Winter 2017, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2017/entries/physicalism/.

Suárez, Mauricio. 2007. "Quantum Propensities." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 38 (2): 418–438. https://doi.org/10.1016/j.shpsb.2006.12.003.

Subbarayappa, B. V. 1971. "The Physical World: Views and Concepts." In *A Concise History of Science In India,* edited by D. M. Bose, S. N. Sen, and B. V. Subbarayappa, 445–483. New Delhi: Indian National Science Academy.

Tegmark, Max. 2007. "Shut up and Calculate." *ArXiv e-prints,* arXiv: 0709.4024.

Teller, Paul. 1998. "Quantum Mechanics and Haecceities." In *Interpreting Bodies: Classical and Quantum Objects in Modern Physics,* edited by Elena Castellani, 114–141. New Jersey: Princeton University Press.

Thomasson, Amie. 2019. "Categories." In *The Stanford Encyclopedia of Philosophy,* Summer 2019, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/sum2019/entries/categories/.

Thomson, J. J. (1928) 2016. *Beyond the Electron.* Cambridge: Cambridge University Press.

Tonomura, Akiro, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa. 1989. "Demonstration of Single-Electron Buildup of an Interference Pattern." *American Journal of Physics* 57 (2): 117–120.

Toraldo di Francia, Giuliano. 1998. "A World of Individual Objects?" In *Interpreting Bodies: Classical and Quantum Objects in Modern Physics,* edited by Elena Castellani, 21–29. New Jersey: Princeton University Press.

Torretti, Roberto. 1978. *Philosophy of Geometry From Riemann to Poincaré.* Dordrecht: D. Reidel Publishing Company.

Van Inwagen, Peter. 1995. *Material Beings.* Cornell University Press. ISBN: 978-0-8014-8306-6. Google Books: mE38_qz5BUAC.

Varzi, Achille. 2019. "Mereology." In *The Stanford Encyclopedia of Philosophy,* Spring 2019, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2019/entries/mereology/.

Wallace, Philip R. 1994. "Comment on "Interference Fringes between Two Separate Lasers," by F. Louradour, F. Reynaud, B. Colombeau, and C. Froehly [Am. J. Phys. 61(3), 242–245 (1993)]." *American Journal of Physics* 62 (10): 950–950. https://doi.org/10.1119/1.17689.

Wang, Kai, Qian Xu, Shining Zhu, and Xiaosong Ma. 2019. "Quantum Wave–Particle Superposition in a Delayed-Choice Experiment." *Nature Photonics,* 1–6. https://doi.org/10.1038/s41566-019-0509-0.

Wasserman, Ryan. 2018. "Material Constitution." In *The Stanford Encyclopedia of Philosophy,* Fall 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/fall2018/entries/material-constitution/.

Weatherson, Brian. 2016. "The Problem of the Many." In *The Stanford Encyclopedia of Philosophy,* Winter 2016, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2016/entries/problem-of-many/.

Weisberg, Michael. 2007. "Three Kinds of Idealization." *The journal of Philosophy* 104 (12): 639–659. https://doi.org/10.5840/jphil20071041240.

Weiskopf, Daniel Aaron. 2008. "The Plurality of Concepts." *Synthese* 169 (1): 145. ISSN: 1573-0964. https://doi.org/10.1007/s11229-008-9340-8.

Wheaton, Bruce R. 1991. *The Tiger and the Shark: Empirical Roots of Wave-Particle Dualism.* Cambridge: Cambridge University Press.

Wiggins, David. 1967. *Identity and Spatio-Temporal Continuity.* Oxford: Basil Blackwell.

Yagisawa, Takashi. 2018. "Possible Objects." In *The Stanford Encyclopedia of Philosophy,* Spring 2018, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2018/entries/possible-objects/.

Zajonc, Arthur. 2008. "Light Reconsidered." In *The Nature of Light: What Is a Photon?,* edited by Chandra Roychoudhuri, A. F. Kracklauer, and Katherine Creath, 3–9. Boca Raton: CRC Press.

Zettili, Nouredine. 2009. *Quantum Mechanics: Concepts and Applications.* John Wiley & Sons, February 17, 2009. ISBN: 978-0-470-02678-6.